Homework #8 PHY523 Fall 2017 Due 11/09/2017

Reading Assignment: Please read Chapter 10, sections 6,7, and 8. Please also read Chapter 11, sections 1-5.

Problems marked with as asterisk are more challenging and/or calculationally involved. Do the other ones first, if you prefer. Come to me or talk to each other if you would like help/hints. I encourage discussion among you, but all submitted work must be your own.

Problem #1 Flatland

Consider a world with only two space dimensions (we denote them as x and y). In this question we develop the rigid body formalism in this world.

(a) In general, how many degree of freedom are there for a rigid body in this world? (Make sure you get this question right. All the rest is based on this one.)

(b) Show that in this world the conserved quantity associated with rotation, which we denote by angular momentum, L, is a scalar (that is, it has only one component).

(c) Show that the angular momentum for a point-like particle is given by

$$L = mrv\sin\phi \tag{1}$$

where ϕ is the angle between r and v.

(d) The angular velocity is $\omega = \dot{\phi}$. Show that the velocity of any point in body can be written as sum of the motion of the center of mass and rotation

$$\vec{v} = \vec{V} + r\omega\hat{\phi} \tag{2}$$

where r is the distance from the center of mass of the body.

(e) Show that the kinetic energy of the body can be written in a form similar to the one we wrote in class, that is, as a sum of movement of the center of mass and rotation

$$T = \frac{MV^2}{2} + \frac{I\omega^2}{2}. \tag{3}$$

In 3d I is a rank two tensor (that is, a matrix). What is I in flatland? Calculate I.

(f) Show that the EOMs for a rigid body are given by

$$m\dot{V} = -\frac{\partial V}{\partial R}, \qquad I\dot{\omega} = -\frac{\partial V}{\partial \phi}. \tag{4}$$

(g) Consider an isolated free body (that is, $V(R,\phi) = 0$.) Explain why in this case the angular velocity is conserved. (Note this is in general true in 2d but not in 3d.)

(h) Consider an isolated free body such that the initial condition are $\phi(0) = \pi/4$ and $\dot{\phi}(0) = a$. Find $\phi(t)$.

Problem #2 Inertia Tensor

A rigid body consists of three equal masses m fastened at the positions $(a,0,0)$, $(0,a,2a)$, and $(0,2a,a)$.

(a) Find the inertia tensor $\mathbf{I}$.

(b) Find the principal moments, along with their associated orthogonal principal axes.

Problem #3
Precession

The earth spins about its axis at an angle of 23° with respect to the unit normal to the plane of the earth's orbit around the sun. In addition, it is observed that the axis of the earth's rotation precesses about the unit normal with a period of about $26,000$ years. This is due to the fact that the earth is an oblate spheroid (the rotation of the earth generates an "equatorial bulge"), and the sun and moon's gravity apply a torque to the spinning earth. From this information, calculate the magnitude of the torque acting on the earth from the sun and the moon. *Hint:* You will need to find the principal moment corresponding to the axis of rotation of the earth. You can assume that the earth is a perfect sphere for this part of the calculation.