Homework #7 PHY523 Fall 2017 Due 10/31/2017

You may slide your HW under my office door (Rm 311) if I am not available in the office when you bring by the HW on Tuesday 10/30.

Problems marked with as asterisk are more challenging and/or calculationally involved. Do the other ones first, if you prefer. Come to me or talk to each other if you would like help/hints. I encourage discussion among you, but all submitted work must be your own.

Problem #1
Angular Momentum

Show that the angular momentum around an axis that goes through the center of mass can be written as

$$L_i = \sum_n m_n \left(x_k x_k \omega_i - x_i x_k \omega_k \right) = I_{ik} \omega_k \tag{1}$$

Problem #2
Rotating Frames

The derivation of the equation of motion (Eq. 9.34) for a rotating frame made the assumptions that the angular velocity $\mathbf{\Omega}$ was constant. Show that if $\dot{\mathbf{\Omega}} \neq 0$, then there is a third "fictitious force" sometimes called the *azimuthal force*, on the right-hand-side of (9.34), equal to $m\mathbf{r} \times \dot{\mathbf{\Omega}}$.

Problem #3
Centrifugal Force

On a certain planet, which is perfectly spherically symmetric, the free-fall acceleration has magnitude $g = g_0$ at the North Pole, and $g = \lambda g_0$ at the equator (λ is a constant between 0 and 1). Find $g(\theta)$, the free-fall acceleration at angle θ with repect to the axis passing through both poles.

Problem #4
Coriolis Force

Repeat the derivation of the equation of motion for the "bead on a rotating wire" (Example 7.6 in the textbook). Instead of using Lagrangian methods, use the expression for the acceleration in a rotating reference frame, Eq. (9.34). Check that your result for the equation of motion agrees with Eq. 7.69. Use a free-body diagram to explain the result for the stationary points of the system.