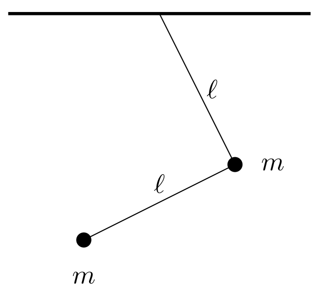
Homework #4 PHY523 Fall 2017 Due 9/26/2017

Problems marked with as asterisk are more challenging and/or calculationally involved. Do the other ones first, if you prefer. Come to me or talk to each other if you would like help/hints. I encourage discussion among you, but all submitted work must be your own.

Problem #1
Double Pendulum



Consider a planar double pendulum (in the x-y plane) in the Earth's gravitational field in the y direction as shown above. The length of both strings is ℓ and both masses are equal, and denoted by m.

(a) How many DOFs are there in this case? Write them in terms of angles.

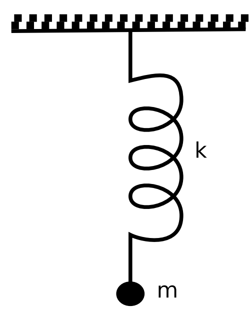
(b) Find the Lagrangian (This is an example Physics GRE question, by the way!)

(c) * Write the EOMs (but do not solve them).

(d) * In the small angle approximation, solve the EOMs.

Problem #2
Spring Pendulum

Consider a planer pendulum that has a spring instead of a wire. (That is, the distance from the support point is not fixed.) The spring constant is k and the spring is considered to be massless. We consider the Earth's gravitational field to work in the y direction



(a) Convince yourself that this system has two DOFs.

(b) Write down the Lagrangian in ϕ and $\lambda \equiv (r - \ell_0)/\ell_0$ coordinates, where ℓ_0 is the equilibrium length of the spring and r is the distance of m from the support point.

(c) * Find the EOM.

(d) * Solve the EOM in the small angle approximation (neglect second order terms in $\lambda, \phi, \dot{\lambda}, \dot{\phi}, \ddot{\phi}, \ddot{\lambda}$). Using

$$\omega_s^2 = \frac{k}{m}, \qquad \omega_p^2 = \frac{g}{\ell_0}, \qquad \tilde{\lambda} = \lambda - \frac{mg}{kl_0}, \tag{1}$$

show that the solution can be written as

$$\tilde{\lambda} = A\cos(\omega_s t + \varphi_s) \qquad \phi = B\cos(\omega_p t + \varphi_p) \tag{2}$$

such that A, B, φ_p and φ_s are the initial conditions. The fact that, in the small angle approximation, the result decouples is very general and we will discuss it in length later in the course.