Homework #3 PHY523 Fall 2017 Due 9/19/2017

Problems marked with as asterisk are more challenging and/or calculationally involved. Do the other ones first, if you prefer. Come to me or talk to each other if you would like help/hints. I encourage discussion among you, but all submitted work must be your own.

Problem #1
The Gradient

For a function f of multiple coordinates, e.g. $f = f(x, y, z)$, then we can express the differential of f as:

$$df = \nabla f \cdot d\mathbf{r} \tag{1}$$

Use this relation to show:

(a) The vector ∇f at any point $\mathbf{r}$ is perpendicular to the surface of constant f through $\mathbf{r}$.
Hint: Choose a small displacement $d\mathbf{r}$ that lies in a surface of constant f. What is df for such a displacement?

(b) The direction of ∇f is at any point $\mathbf{r}$ is the direction in which f increases tastest as we move away from $\mathbf{r}$.
Hint: choose a small displacement $d\mathbf{r} = \epsilon\mathbf{u}$ where $\mathbf{u}$ is a unit vector and ϵ is small. Find the direction of $\mathbf{u}$ for which the corresponding df is maximum, remembering that $\mathbf{a} \cdot \mathbf{b} = ab\cos\theta$.

Problem #2
Scattering

Consider an elastic collision of two particles as in Example 4.8 (Page 143 of the textbook), but with unequal masses: $m_1 \neq m_2$. Show that the angle θ between the two outgoing velocities satisfies $\theta < \pi/2$ if $m_1 > m_2$, but $\theta > \pi/2$ if $m_1 < m_2$.

Problem #3
Rigid Bodies

A rigid body can be defined as a collection of point-like particles such that the distances between all the particles are constant.

(a) Show that in general a rigid body has 6 DOFs (degrees of freedom). Find a system of coordinates to parameterize these 6 DOFs, that is, find a way to conveniently describe the state of an aribitrary rigid body.

(b) Can you think of a rigid body that has less than 6 DOFs? If yes give an example and show how many DOFs it has.

(c) Can you think of a rigid body that has more than 6 DOFs? If yes give an example and show how many DOFs it has.

(d) Consider a hypothetical world where there are four space dimensions. How many DOFs a rigid body in such a world would have? Can you find how many DOFs a rigid body has in an n-dimensional world?

Problem #4
Off-Road Resonance

When a car drives along a "washboard" road, the regular bumps cause the wheels and axles to oscillate on the springs. Your goal is to estimate at which speed the bumps will have the worst effect on the driving experience, i.e. when the bumps on the road drive the system at its resonance frequency.

(a) When 4 80 kg people get in the car, the body sinks by 2 cm. Use this data to estimate the spring constants of the assembly.

(b) If the axle assembly (axle plus wheels) has a total mass of 50 kg, what is the natural angular frequency ω of the assembly oscillating on its' two springs?

(c) If the bumps on the road are 80 cm apart, at about what speed would these oscillations go into resonance?

Problem #5
Computation

In the week 2 notebook, calculate the area swept out per unit time and plot it as a function of time. It should be constant, as dictated by Kepler's law. Recall that the inverse square law played no role in the derivation of Kepler's law, only conservation of angular momentum. Change the force law to a Hook law spring, $\vec{F} = k\vec{r}$, and check that the area law still holds.