Homework #1 PHY523 Fall 2017 Due 9/5/2017

Problems marked with as asterisk are more challenging and/or calculationally involved. Do the other ones first, if you prefer. Come to me or talk to each other if you would like help/hints. I encourage discussion among you, but all submitted work must be your own.

Problem #1
Energy Equations

Show that for a single particle with constant mass, the equations of motion implies the following differential equation for the kinetic energy, T:

$$\frac{dT}{dt} = \mathbf{F} \cdot \mathbf{v}, \tag{1}$$

while if the mass varies, one obtains

$$\frac{d(mT)}{dt} = \mathbf{F} \cdot \mathbf{p}, \tag{2}$$

Problem #2*
Rocket Ship!

Rockets are propelled by the momentum reaction of the exhaust gases expelled from the tail. Since these gases arise from the reaction of the fuels carried in the rocket, the mass of the rocket is not constant, but decreases as the fuel is expended. Show that the equations of motion for a rocket projected vertically upward in a uniform gravitational field, neglecting atmospheric resistance, is

$$m\frac{dv_{\text{rocket}}}{dt} = -v_{\text{gas}}\frac{dm}{dt} - mg \tag{3}$$

where m is the (non-constant) mass of the rocket and v_{gas} is the (constant) velocity of the gases relative to the rocket. Integrate this equation to obtain v as a function of m, assuming a constant time rate of loss of mass. Show, for a rocket starting initially from rest, with $v_{\text{gas}} = 2100\text{m/s}$ and a mass loss per second equal to 1/60th of the initial mass, that in order to reach the escape velocity the ratio of the weight of the fuel to the weight of the empty rocket must be almost 300!

Problem #3
Separation of Variables

Some one-dimensional problems can be reduced to the problem of performing an integral. One such case is when there is a force that depends only on a particle's velocity: $F = F(v)$.

(a) Write down Newton's second law and separate the variables by rewriting it as

$$m\frac{dv}{F(v)} = dt \tag{4}$$

Now integrate both sides of this equation and show that

$$t = m\int_{v_0}^{v} \frac{dv'}{F(v')}. \tag{5}$$

Provided that this integral is do-able, this gives t as a function of v, which can then be inverted to give v as a function of t.

(b) Use this method on the following problem: A mass m has veloicty v_0 at time $t = 0$, and coasts along the x axis in a medium where there is a drag force given by $F(v) = -cv^{3/2}$. Find v in terms of t and the other given parameters. At what time t_f (if any) will the mass come to rest?

Problem #4
Weird 1D Motion

The force acting on a particle of mass m, constrained to move along a straight line, is given by:

$$F = kvx, \tag{6}$$

where k is a constant, v is the instantaneous velocity, and x is the position. For initial conditions at $t = 0$, we have $v = v_0$, and $x = 0$. Find the expression for the position as a function of time.

Problem #5* — Rotating Frames

Recall a fundamental tenet of inertial reference frames: if there is zero net force on a system, its center of mass is constrained to move in a straight line. This would be the case (for example) for a frictionless system of a puck sliding on a turntable rotating at constant angular velocity ω- the rotation of the table is irrelevant, since it does not apply any force to the moving puck. However, consider the same system from the standpoint of an observer tied down to the rotating turntable. Describe qualitatively the puck's path as observed by this person.

Now let's take a quantitative approach:

(a) Write down the polar coordinates r, ϕ of the puck as functions of time, as measured in the inertial frame $\mathcal{S}$ of an observer on the stationary ground. (Assume that the puck was launched along the axis $\phi = 0$ at $t = 0$.

(b) Now write down the polar coordinates r', ϕ' of the puck as measured by an observer (frame $\mathcal{S}'$) at rest on the turntable. (Choose the coordinates so that ϕ and ϕ' coincide at $t = 0$.) Describe and sketch the path seen by this second observer. Is the frame $\mathcal{S}'$ inertial?

Problem #6 — Constant Speed

Prove that if $\mathbf{v}(t)$ is any vector which depends on time, but which *has constant magnitude* $|\mathbf{v}(t)|$, then $\dot{\mathbf{v}}(t)$ is *perpendicular* to $\mathbf{v}(t)$. Prove the converse as well (that orthogonal $\dot{\mathbf{v}}(t)$ and $\mathbf{v}(t)$ imply constant $|\mathbf{v}(t)|$).

Problem #7 — Computation

For an N-body simulation, we presume a $1/r$ potential between masses m_i, where i ranges from 1 to N. The potential energy between masses m_i and m_j is

$$U(\vec{r}_i, \vec{r}_j) = -\frac{m_i m_j}{r_{ij}} \tag{7}$$

where $r_{ij} = |\vec{r}_i - \vec{r}_j|$. Show that the force on mass i due to j is

$$\vec{F}_{ij} = \frac{\vec{r}_j - \vec{r}_i}{r_{ij}^3} \tag{8}$$

Now edit the mathematica code for week 1 to include 3 bodies. Also toy around with the code...e.g. try a very heavy mass for one of them, to mock up the solar system, try different initial conditions, etc. Take note of observations for the 3 body code!