Lecture 08.2 - Physics 523

Recall Newton's 2nd Law in rotating frames

$$m\ddot{\vec{r}} = \vec{F} + \underbrace{2m\dot{\vec{r}}\times\vec{\Omega}}_{\text{Coriolis force}} + \underbrace{m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}}_{\text{Centrifugal force}}$$

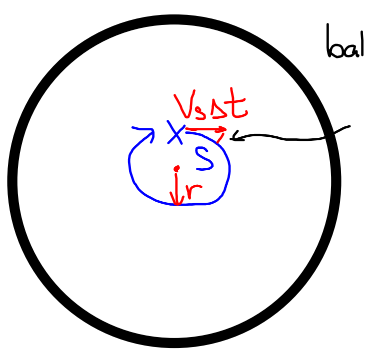
- ★ Coriolis force is non-zero only for non-zero velocities
- ★ Centrifugal force → note that $(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$ is <u>always</u> $\perp$ to $\vec{\Omega}$ and in the $\vec{\Omega} \leftrightarrow \vec{r}$ plane

Making sense of the forces: Consider $\vec{F}=0$ → trajectory in inertial frame is a <u>straight line</u>

In S_0 frame

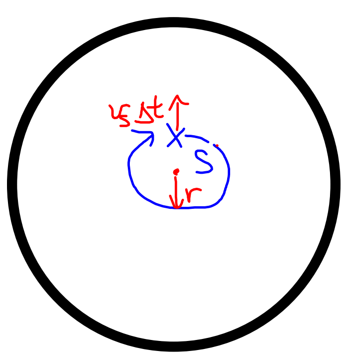
$v_S = 0$

$v_{\tilde{S}} = V_S$



ball has acceleration relative to frame S — velocity has changed!

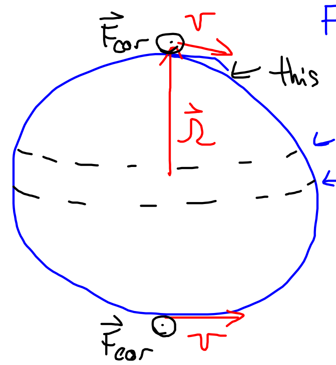
change in r due to centrifugal accel.



Velocity of turntable is <u>faster</u> at new position! $\vec{v}_{new} - \vec{v}_{old} = \Omega(r + v_S\Delta t) - \Omega(r) = \Omega v_S \Delta t\,(-\hat{\theta})$

$\Rightarrow \vec{a} = \Omega v_S(-\hat{\theta})$

Similar on earth → at equator, velocity is fastest on surface, slower at points in Northern / Southern hemispheres

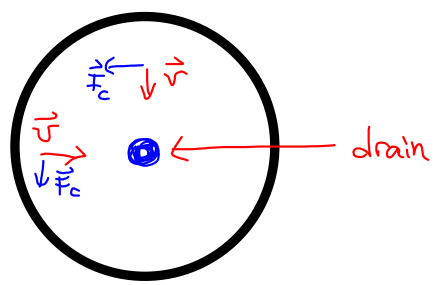


Fire projectile due south from N-pole (centrifugal force is vanishing)

* Aside → on NH, $\vec{F}_{cor}$ is to the rt. of projectile
SH, to the left

Toilets

On N-pole



Net velocity towards drain induces CCW accel.! Same dir. for entire N-hemisphere

On S-pole (or below equator)

Not true $\vec{F}_{cor}$ is a tiny effect compared to initial $\vec{L}$ of the system

Can use this, but need to carefully control state of the water / fluid

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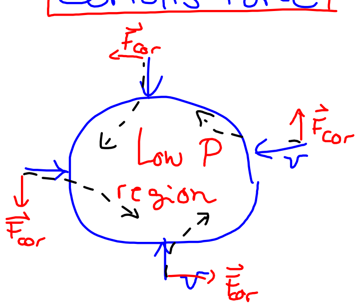
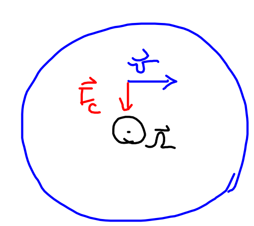
Coriolis Force This is important when velocities are large or the time-scales are large

large storm systems, ocean currents



Storms in NH swirl CCW

Ocean currents follow element of fluid near N-pole



Ocean currents move in CW dir on N-hemisphere

CCW on southern hemisphere

Cute aside Remember the magnetic force law $\vec{F}_B = q\vec{v} \times \vec{B}$ $\vec{F}_{cor} = m\vec{v} \times (2\vec{\Omega})$ No significance, but handy sometimes

Example Felix Baumgartner's jump $m\ddot{\vec{r}} = m\vec{g}_0 + \vec{F}_{cor} + \vec{F}_{CF} + \vec{F}_{other}$

Assume he did it at equator, and reached terminal velocity of 800 mph (↙ 360 m/s) for 2 min

Q1 does centrifugal force play a role? A. No → @ terminal velocity, $\vec{F}_{CF} + \vec{F}_{grav} + \vec{F}_{drag} = 0$

Q2 Which dir is coreolis force? A. East

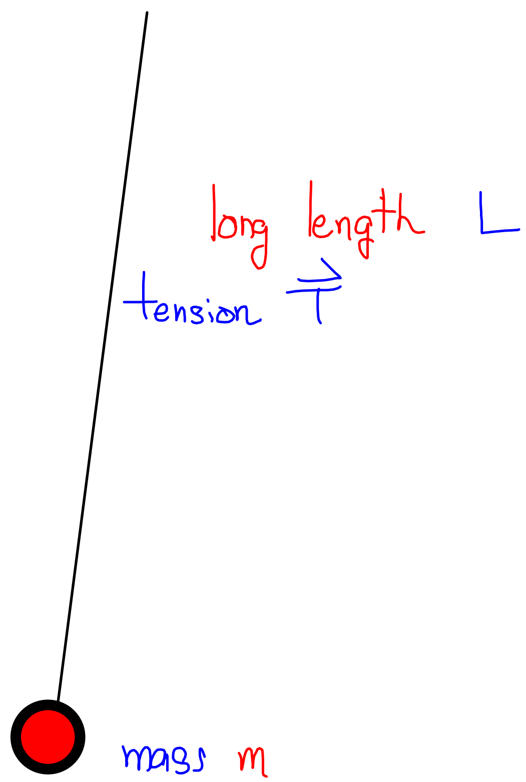
Q3 magnitude? A. $\vec{F}_{cor} = m\vec{v}\times(2\vec{\Omega}) = (100kg)(360m/s)(2\cdot 7\times10^{-5} rad/s)(\hat{east}) \sim (5\times10^{-2} m/s^2)(100kg) = 5$ Newtons

($360m/s \simeq 500$, $2\cdot7\times10^{-5} \simeq 10^{-4}$ r/s)

Q4 deviation? A. $\Delta x = \frac{1}{2}at^2 = \frac{1}{2}(5\times10^{-2} m/s^2)(120s)^2 \sim 2.5\times10^2 m$ Not negligible!

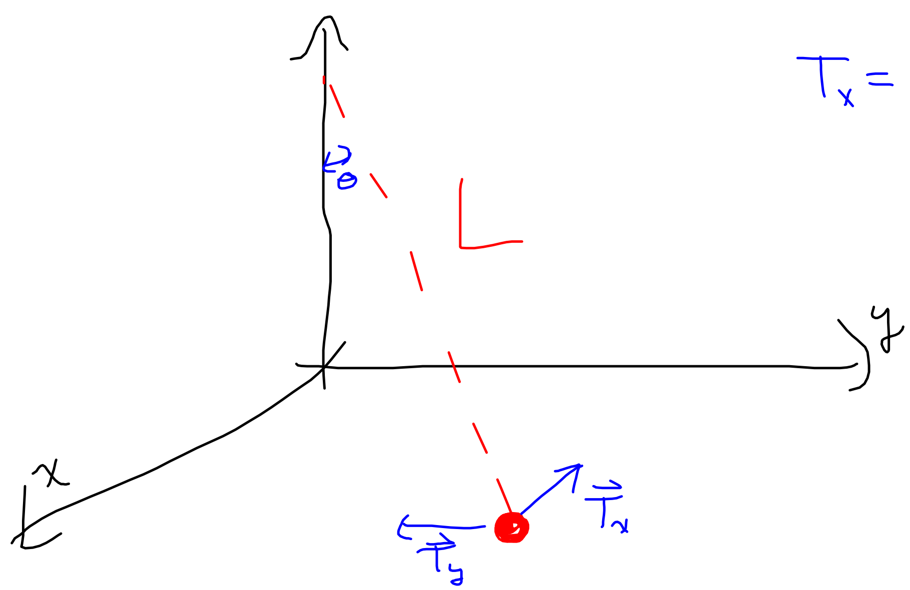
($120s \simeq (100s)$)

Focault Pendulum



$$m\ddot{\vec r} = \underbrace{\vec T + m\vec g_0 + 2m\dot{\vec r}\times\vec\Omega + m(\vec\Omega\times\vec r)\times\vec\Omega}_{\text{combine to give effective free-fall accel.}} = \vec T + m\vec g + 2m\dot{\vec r}\times\vec\Omega$$

for large L, small oscillations, $\vec T = -mg\,\hat L$ ← dir. along wire



$T_x = -mg\,\frac{x}{L}$ $\quad T_y = -mg\,\frac{y}{L}$

$$\ddot x = -g\,\frac{x}{L} + 2\dot y\,\underbrace{\Omega\cos\theta}_{\Omega_z}$$

← angle between Ω & $\hat z$

$$\ddot y = -\frac{gy}{L} - 2\dot x\,\underbrace{\Omega\cos\theta}_{\Omega_z}$$

$\frac{g}{L} = \omega_0^2$

$\ddot x - 2\Omega_z\dot y + \omega_0^2 x = 0$

$\ddot y + 2\Omega_z\dot x + \omega_0^2 y = 0$

$\eta \equiv x + iy$ $\qquad \ddot\eta + 2i\Omega_z\dot\eta + \omega_0^2\eta = 0$ $\qquad$ guess $\eta = Ae^{-i\alpha t}$

$-\alpha^2 + 2\Omega_z\alpha + \omega_0^2 = 0$ $\qquad \alpha = \Omega_z \pm \sqrt{\Omega_z^2 + \omega_0^2} \simeq \Omega_z \pm \omega_0$

$\eta(t) = e^{-i\Omega_z t}\left(c_1 e^{-i\omega_0 t} + c_2 e^{+i\omega_0 t}\right)$ $\quad$ @ $t=0$, set $x=A$, $y=0$ ⇒ $c_1 = c_2 = \frac{A}{2}$ $\quad \eta(t) = Ae^{-i\Omega_z t}\cos(\omega_0 t)$

As time goes by, x motion turns into y motion!