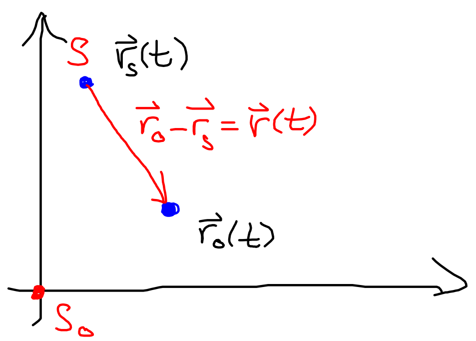
Lecture 07.1 - Physics 523

Announcements Midterm exam next tuesday, in Lecture! BE THERE!

Non-inertial Frames Recall last week we discussed physics in 2 different frames: S_o & S
inertial non-inertial

Classical mech, in frame S_o obeys Newton's Laws $\vec{F} = m\ddot{\vec{r}}_o$. If frame S_o sees trajectory $\vec{r}_o(t)$, & frame S moves through trajectory $\vec{r}_s(t)$, then S sees trajectory $\vec{r}_o - \vec{r}_s \equiv \vec{r}(t)$



$$\vec{F} = m\ddot{\vec{r}}_o = m(\ddot{\vec{r}} + \ddot{\vec{r}}_s)$$

$$m\ddot{\vec{r}} = \vec{F} - m\ddot{\vec{r}}_s$$

$$\boxed{m\ddot{\vec{r}} = \vec{F} - m\vec{A}}$$

acceleration of N-I frame S

We can now utilize this formula for addressing physics as observed in accelerating reference frames!

Tidal Forces

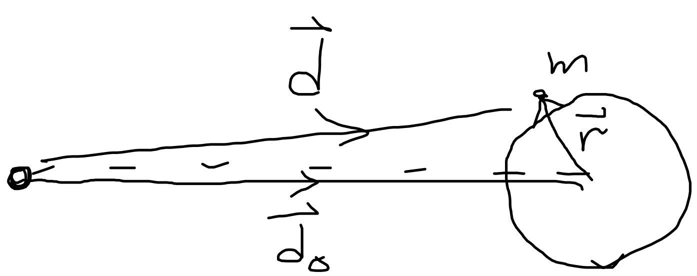
The earth's oceans respond to the relative orientations of the earth + moon

Forces on mass m on surface of earth

1) $-mg\hat{r}$ earths center is origin

2) $F_{moon} = -\frac{GM_0 m}{d^2}\hat{d}$

3) Other local forces, $\vec{F}_{ng}$



Earth has centripetal accel, $\vec{A} = -\frac{GM_0}{d_0^2}\hat{d}_0$

Using non-inertial frame mod. of Newton's 2nd $m\ddot{\vec{r}} = \vec{F} - m\vec{A}$: $m\ddot{\vec{r}} = -mg\hat{r} - \frac{GM_0 m}{d^2}\hat{d} + \vec{F}_{ng} + \frac{GM_0 m}{d_0^2}\hat{d}_0$

$$\boxed{m\ddot{\vec{r}} = -mg\hat{r} + \vec{F}_{ng} + \vec{F}_{tidal}}$$

$$F_{tidal} = -GM_0 m\left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_0}{d_0^2}\right)$$

Equipotential Surface

Claim: The oceans surface is an equipotential surface

Proof for conservative forces $\vec{F} = -\vec{\nabla} U$ for $\hat{n}$ on surface, $\vec{F}\cdot\hat{n} = -(\vec{\nabla} U)\cdot\hat{n}$ if surface is in equilibrium, $\vec{F}\cdot\hat{n} = 0$ (no lateral motion of water)

$\boxed{\text{So } \vec{\nabla} U \text{ vanishes along surface} \Rightarrow \text{equipotential}}$

$\vec{F}_{tidal} = -GM_0 m\left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_0}{d_0^2}\right)$ like the gradient of 2 terms 1) $\sim -\frac{1}{d}$ 2) $-\frac{x}{d_0^2}$

$$U_{tidal} = -GM_0 m\left(\frac{1}{d} + \frac{x}{d_0^2}\right)$$
$$= -\frac{GM_0 m}{d_0}\left(\frac{d_0}{d}\right)$$

$$U_{tot} = mgh - GM_0 m\left(\frac{1}{d} + \frac{x}{d_0^2}\right) \overset{\text{low tide}}{=} -\frac{GM_0 m}{d_0}\left(\frac{d_0}{d}\right) = -\frac{GM_0 m}{d_0}\left(\frac{d_0}{\sqrt{R_e^2 + d_0^2}}\right) \overset{\text{taylor exp}}{\simeq} -\frac{GM_0 m}{d_0}\left(1 - \frac{1}{2}\left(\frac{R_e}{d_0}\right)^2\right)$$

h: height above low tide

$$= mgH - \frac{GM_0 m}{d_0}\left(\frac{d_0}{d} + \frac{x}{d_0}\right) = mgH - \frac{GM_0 m}{d_0}\underbrace{\left[\frac{d_0}{d_0 \pm R_e} \pm \frac{R_e}{d_0}\right]}_{\text{1st order term cancels}}$$

$$\simeq mgH - \frac{GM_0 m}{d_0}\left[1 + \left(\frac{R_e}{d_0}\right)^2\right]$$

$\boxed{mgH = \frac{GM_0 m}{d_0}\cdot\frac{3}{2}\frac{R_e^2}{d_0^2}}$ and $g = \frac{GM_e}{R_e^2}$ $\boxed{H = \frac{3}{2}\frac{M_0}{M_e}\frac{R_e^4}{d_0^3}}$

All this came from $\boxed{m\ddot{\vec{r}} = \vec{F} - m\vec{A}}$

$\boxed{\text{Example}}$ Spinning w/ a coffee cup

angular frequency ω

$m\ddot{\vec{r}} = \vec{F} - m\vec{A}$ $\quad \vec{F} = m\vec{g} + \vec{F}_{ng}$; $\vec{A} = -\frac{v^2\hat{r}}{r}$

$m\ddot{\vec{r}} = m\vec{g} + \frac{mv^2}{r}\hat{r} + \vec{F}_{ng} = -\vec{\nabla}U_{eff} + \vec{F}_{ng}$

$-\vec{\nabla}U_{eff} = m\vec{g} + \frac{mv^2}{r}\hat{r} = m\vec{g} + m\omega^2 r\,\hat{r}$

surface of liquid is equipotential

$U_{eff} = mgh - m\omega^2 r^2 = \text{const} = 0$ (h=0 for r=0 convention)

$$\boxed{h = \frac{\omega^2 r^2}{g}}$$