HW#3 Solutions

1) a) Show $\vec{\nabla} f$ is $\perp$ to surface of constant f through $\vec{r}$

$d_\lambda f = 0$, for λ along dir. of constant f. $\quad d_\lambda f = (\vec{\nabla} f)\cdot d_\lambda \vec{r} = 0 \Rightarrow \vec{\nabla} f \perp$ to $d_\lambda \vec{r}$

b) Show $\vec{\nabla} f$ is in dir. of fastest variation in f $\quad d\vec{r} = \varepsilon \hat{u}$, $d_u f = \vec{\nabla} f \cdot (\varepsilon \hat{u}) = \varepsilon |\vec{\nabla} f| \cos\theta$ is maximized for $\theta = 0 \rightarrow df$ is biggest along dir. of $\vec{\nabla} f$.

2) Momentum cons: $m_1 \vec{v}_1 = m_1 \vec{v}_1{}' + m_2 \vec{v}_2{}'$

Energy cons: $\frac{1}{2} m \vec{v}_1^{\,2} = \frac{1}{2} m_1 \vec{v}_1{}'^2 + \frac{1}{2} m_2 \vec{v}_2{}'^2$

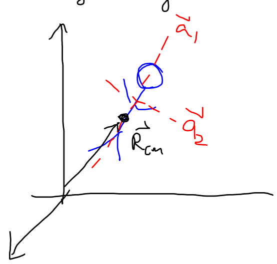
$$\frac{1}{2} m_1 \vec{v}_1^{\,2} = \frac{1}{2}\frac{(m_1\vec{v}_1)^2}{m_1} = \frac{1}{2m_1}\left(m_1\vec{v}_1{}' + m_2\vec{v}_2{}'\right)^2 = \frac{1}{2} m_1 \vec{v}_1{}'^2 + \frac{1}{2} m_2 \vec{v}_2{}'^2$$

$$\hookrightarrow \frac{1}{2m_1}\left(m_1^2 \vec{v}_1{}'^2 + m_2^2 \vec{v}_2{}'^2 + 2 m_1 m_2 \vec{v}_1{}' \cdot \vec{v}_2{}'\right)$$

$$\vec{v}_1{}' \cdot \vec{v}_2{}' = \frac{1}{2} m_2 \vec{v}_2{}'^2 \left(1 - \frac{m_2}{m_1}\right) \begin{cases} < 0 & m_2 > m_1 \\ > 0 & m_2 < m_1 \end{cases}$$

$$\boxed{\begin{array}{l} \cos\theta_{12}' < 0 \quad m_2 > m_1 \rightarrow \theta_{12}' > \pi/2 \\ \cos\theta_{12}' > 0 \quad m_2 < m_1 \rightarrow \theta_{12}' < \pi/2 \end{array}}$$

3) a) A rigid body has a CM, and an orientation (eg. action figure)



$\vec{R}_{cm}$ is 3 DoF

orientation of $\vec{a}_1$ w.r.t. coord. axes (2 angles)

orientation of $\vec{a}_2$, one more angle

3 total cart., 3 angles

6 d.o.f.

b) If all particles lie along line, then there is no $\vec{a}_2$ vector $\Rightarrow$ 3 cart. coord., & only 2 angles $\rightarrow$ 5 d.o.f.

Similarly, a point-particle has no "orientation" so only 3 d.o.f. there

c) No. The only way to further increase the # of d.o.f. is to add internal d.o.f., but then the body is not "rigid"

d) There are $\boxed{10 \text{ d.o.f.}}$ in this case $\quad$ 4 total for $\vec{R}_{cm}$, 6 angles

$\hookrightarrow$ 6 planes through object, $\quad x_1\text{-}x_2 \quad x_2\text{-}x_3 \quad x_3\text{-}x_4 \quad x_1\text{-}x_3 \quad x_2\text{-}x_4 \quad x_1\text{-}x_4$

for n dimensions, n for CM, $\binom{n}{2} = \frac{n(n-1)}{2}$ planes (angles) $\quad \boxed{\text{total } \frac{n(n+1)}{2} \text{ dof's}}$

4) a) $F = -kx$, $\quad F = -mg = -(320\,\text{kg})(10\,\text{N/kg}) = -(3200\,\text{N}) = -4k(.02\,\text{m})$ $\quad \boxed{k = 4\times10^4\ \text{N/m}}$

$\uparrow$ 4 springs, one for each wheel

b) $m\ddot{x} = -kx$, and $\omega^2 = k/m$ $\quad$ there are 2 springs on each assembly $\quad k = 8\times10^4\ \text{N/m}$

$$\boxed{\omega = \sqrt{\frac{8\times10^4\ \text{N/m}}{50\,\text{kg}}} = 40\ \text{rad/s}}$$

c) $\omega = 2\pi f = 2\pi\left(\frac{v}{.8\,\text{m}}\right) = 40\ \text{rad/s}$ for $\quad \boxed{v = 5\ \text{m/s} \sim 11.2\ \text{mph}}$