HW#1 Solutions

1) $$\frac{dT}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}m\vec{v}^2\right) = \frac{d}{dt}\left(\frac{\vec{p}^2}{2m}\right) \overset{\text{constant } m}{=} \frac{\dot{\vec{p}}\cdot\vec{p}}{m} = \vec{F}\cdot\left(\frac{\vec{p}}{m}\right) = \vec{F}\cdot\vec{v}$$

$$\frac{d(mT)}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}(m\vec{v})^2\right) = \frac{d}{dt}\,\tfrac{1}{2}\vec{p}^2 = \dot{\vec{p}}\cdot\vec{p} = \vec{F}\cdot\vec{p}$$ (m did not appear explicitly, so not necc. fixed!)

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2) Easiest way to approach this is to keep track of momentum $p = \underbrace{p_{gas}}_{\text{expelled gas}} + p_{rocket} = m_{gas} v_{gas}^{avg} + m_{rock} \cdot v_{rock}$

Look at small change in momentum due to mass loss + velocity change:

$\uparrow v_{rock}$

$\downarrow v = v_{rock} - v_{gas}$ (velocity of the bit of gas just expelled)

$$\Delta p = \underbrace{\Delta m_{gas}}_{= -\Delta m_{rock}} (v_{rock} - v_{gas}) + \Delta m_{rock} v_{rock} + m_{rock} \Delta v_{rock}$$

$$\frac{\Delta p}{\Delta t} \to \frac{dp}{dt} = v_{gas}\frac{dm}{dt} + m\frac{dv}{dt} = \vec{F} = -mg \Rightarrow \boxed{m\frac{dv}{dt} = -v_{gas}\frac{dm}{dt} - mg}$$

Solve it: $dv = -v_{gas}\dfrac{dm}{m} - g\,dt$

$$\int_{v_0=0}^{v} dv = -v_{gas}\int_{m_0}^{m}\frac{dm}{m} - g\int_{t_0=0}^{t} dt \to \boxed{v = -v_{gas}\ln\frac{m}{m_0} - gt}$$

Say mass loss rate is n, then $m = m_0 - nt$, or $t = \dfrac{m_0 - m}{n}$ $\Rightarrow v = -v_{gas}\ln\dfrac{m}{m_0} - g\left(\dfrac{m_0-m}{n}\right)$

Now plug in #'s: $v_{esc} = 11{,}200\ \mathrm{m/s}$, $v_{gas} = 2100\ \mathrm{m/s}$ $\quad n = \dfrac{m_0}{60\,\mathrm{s}}$ $\quad (11{,}200\ \mathrm{m/s}) = (2100\ \mathrm{m/s})\ln\left(\dfrac{m_0}{m}\right) - (9.8\ \mathrm{m/s^2})(60\,\mathrm{s})\left(1 - \dfrac{m}{m_0}\right)$

$\boxed{\text{Sol'n} \quad \dfrac{m_0}{m} \simeq 295}$

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3) a) $\vec{F} = m\ddot{\vec{r}} = m\dot{\vec{v}}$ in 1D, this is $F = m\dot{v}$, and if F is purely a function of v: $F = F(v)$, have

$$F(v) = m\frac{dv}{dt} \Rightarrow \boxed{\frac{dv}{F(v)} = \frac{dt}{m}}$$ (If m is function of t) or $\boxed{m\dfrac{dv}{F(v)} = dt}$ (m may be function of v)

Integrate both sides (m constant) $\boxed{m\displaystyle\int_{v_0}^{v}\frac{dv}{F(v)} = \int_{t_0=0}^{t} dt = t}$

b) $F(v) = -cv^{3/2}$ $\qquad -m\displaystyle\int_{v_0}^{v}\frac{dv}{cv^{3/2}} = 2m\left.\frac{1}{v^{1/2}}\right|_{v_0}^{v} = \frac{2m}{c}\left(\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_0}}\right) = t$ or $\boxed{v = \left[\dfrac{ct}{2m} + \dfrac{1}{\sqrt{v_0}}\right]^{-2}}$ need $t_f \to \infty$ to stop the mass

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4) $F = kvx = \frac{1}{2}k\frac{d}{dt}(x^2)$ $\qquad m\ddot{x} = \frac{1}{2}k\dot{(x^2)}$ Integrate once: $m\dot{x} = \frac{1}{2}kx^2 + C$ use intal cond. $x_0 = 0$, $v = v_0 \Rightarrow C = mv_0$

$m\dot{x} = \frac{1}{2}kx^2 + mv_0$ and then integrate again after separating variables

$$\frac{dx}{\frac{k}{2m}x^2 + v_0} = dt \to t = \int_{x_0=0}^{x}\frac{dx}{\frac{k}{2m}x^2 + v_0} = \frac{\tan^{-1}\left(\sqrt{\frac{k}{2mv_0}}\,x\right)}{\sqrt{\frac{kv_0}{2m}}} \text{ or } \boxed{x = \sqrt{\frac{2mv_0}{k}}\tan\left(\sqrt{\frac{kv_0}{2m}}\,t\right)}$$

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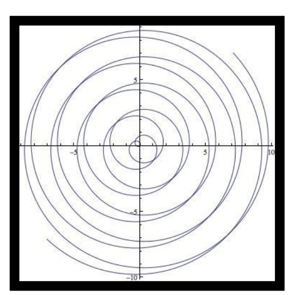
5) Describe motion: say the puck goes through the center of the turntable, then to the observer on the table, the puck rotates about the origin (the turntable center being a convenient choice) at angular frequency ω, while the distance of the puck to the center varies linearly w/ time.

freq. of turntable

a) For observer S, $\phi = \text{const}$ (but shifts to $\phi + \pi$ when/if puck passes through origin) $r = r_0 + vt$

b) for observer S', $\phi_{puck} = \phi_0 \overset{\text{because } S' \text{ observer moves in } +\phi \text{ dir. in } S\text{-coord.}}{-} \omega t$, $r = r_0 + vt$

to plot, $x = r\cos\phi$ $\quad y = r\sin\phi$



6) $|\vec{v}(t)| = \text{const} \Rightarrow \vec{v}(t)\cdot\vec{v}(t) = \text{const} \Rightarrow \frac{d}{dt}\vec{v}\cdot\vec{v} = 2\vec{v}\cdot\dot{\vec{v}} = 0 \Rightarrow \boxed{\vec{v} \text{ is } \perp \text{ to } \dot{\vec{v}}}$

neither is 0 for general $v(t)$

Converse $0 = \vec{v}\cdot\dot{\vec{v}} = \frac{1}{2}\frac{d}{dt}(\vec{v}\cdot\vec{v}) = 0$ if $\vec{v}$ is $\perp$ to $\dot{\vec{v}}$