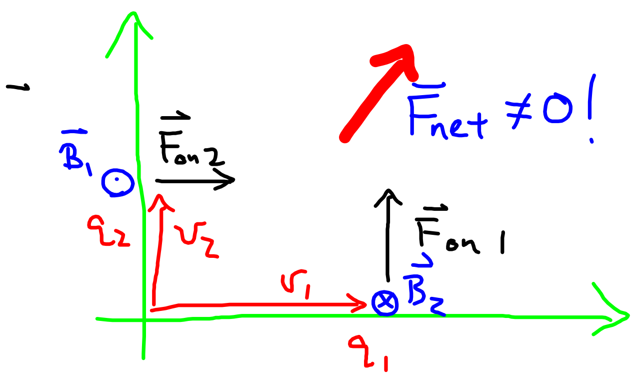
Lecture 01.2 - Physics 523

Note B-forces seem to violate Newton's 3rd → patched up by accounting for momentum carried by EM waves given off by accelerating charges



Drag forces

Air resistance, viscous drag, lift

lift: force ⊥ to motion

→ falling objects & terminal velocity

In many cases F_{drag} is in opposite direction of velocity: $\vec{f}(\vec{v}) = -f(v)\,\hat{v}$

$f(v)$: can be very complicated (turbulence, etc.)

$\hat{v}$: unit vector in dir. of $\vec{v}$

Taylor expand $f(v) = bv + cv^2 + \mathcal{O}(v^3)$

bv: viscous drag (jelly-like resistance)

cv^2: ↳ air type resistance

for air can usually neglect b, for jelly, can usually neglect c (see example 2.1)

Linear drag: $F = m\dot{v} = -bv$ $\quad \dot{v} = -\frac{b}{m}v \Rightarrow \boxed{v = v_0 e^{-\frac{b}{m}t}}$ Motion slows exponentially

How did we get that? $\quad \dot{v} = -\frac{b}{m}v$ or $\frac{dv}{dt} = -\frac{b}{m}v$ or $\boxed{\frac{dv}{v} = -\frac{b}{m}dt}$ ← "separation of variables" Integrate both sides

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{b}{m}\int_{t_0=0}^{t} dt \qquad \boxed{\ln\frac{v}{v_0} = -\frac{b}{m}(t - t_0)} \qquad \boxed{\frac{v}{v_0} = e^{-\frac{b}{m}t}}$$

This technique comes in quite handy whenever F is purely a function of v (always get an integral)

2 Dimensional Motion

$m\ddot{\vec{r}} = \vec{F}_{ext} - f(v)\hat{v}$

Look at 2 cases: 1) Linear Drag $m\dot{\vec{v}} = \vec{F}_{ext} - bv\hat{v} = \vec{F}_{ext} - b\vec{v}$

$m\dot{v}_x = F_x^{ext} - bv_x$ & $x \leftrightarrow y$

Each eq. depends only on v_x or v_y separately

2) Quadratic Drag $m\dot{\vec{v}} = \vec{F}_{ext} - cv^2\hat{v} = \vec{F}_{ext} - c\sqrt{v_x^2+v_y^2}\,\vec{v}$

No longer the case! 2 "Coupled" diff. eq. (HARD)

1 Dimension at a time w/ linear drag (Scuba diver's softball league)

1) "Ground ball" – normal force balances gravity $m\dot{v}_x = -bv_x \Rightarrow v_x = v_x^0 e^{-\frac{b}{m}t}$

2) "Pop fly" – $m\dot{v}_y = -mg - bv_y \rightarrow \dfrac{dv_y}{g+\frac{b}{m}v_y} = -dt \rightarrow \displaystyle\int_{v_0}^{v}\frac{dv_y}{g+\frac{b}{m}v_y} = \frac{m}{b}\log\left(g+\frac{b}{m}v_y\right)\Big|_{v_0}^{v_y} = -t$

$$\frac{g+\frac{b}{m}v_y}{g+\frac{b}{m}v_0} = e^{-\frac{b}{m}t} \rightarrow v_y = \frac{m}{b}\left[\left(g+\frac{b}{m}v_0\right)e^{-\frac{b}{m}t} - g\right]$$

★ Note $t\to\infty$ gives $v_f \simeq -\frac{gm}{b}$ (terminal v)

★ Because v_x and v_y eq's are un-coupled, can put these two results together for any initial conditions

$$\boxed{\vec{v}(t) = \hat{e}_1\left[v_x^0 e^{-\frac{b}{m}t}\right] + \hat{e}_2\left[\frac{gm}{b}\left(e^{-\frac{b}{m}t}-1\right) + v_y^0 e^{-\frac{b}{m}t}\right]}$$

Can also integrate this to get $\vec{r}(t)$:

$$y = y_0 - \left[g\frac{m^2}{b^2}\left(e^{-\frac{b}{m}t} - 1 + \frac{b}{m}t\right) + v_y^0\frac{m}{b}\left(e^{-\frac{b}{m}t}-1\right)\right]$$

★ $\frac{m}{b}$ must have units of time (arg. of exp. must be dimensionless) $\tau = \frac{m}{b}$ is a "characteristic time" of this physical system

Quadratic air resistance

Moving through a relatively rarefied substance like air, most objects experiance quadratic drag

$\vec{f}_{drag} = -cv^2\hat{v} = -cv\vec{v}$ $\quad m\ddot{\vec{r}} = m\dot{\vec{v}} = \vec{F}_{ext} - cv\vec{v}$

linear quadratic

non-linear diff eq. ☹

But the 1D case is integrable!

1D quadratic drag forces

"Runaway train" No $\vec{F}_{ext}$:

careful w/ signs assume v_x positive

$\dot{v}_x = -\frac{c}{m}v_x^2 \rightarrow \frac{dv}{v_x^2} = -\frac{c}{m}dt \rightarrow \int_{v_x^o}^{v_x}\frac{dv_x}{v_x^2} = -\frac{c}{m}\int_{t_0}^{t}dt \rightarrow \frac{1}{v_x^o} - \frac{1}{v_x} = -\frac{c}{m}(t-t_0)$

$\boxed{v_x = \left[\frac{c}{m}t + \frac{1}{v_x^o}\right]^{-1}}$ or $v_x = \dfrac{v_x^o}{1+t/\tau}$ for $\tau = \dfrac{m}{cv_x^o}$ Integrate to get $\boxed{x(t) = x_0 + v_x^o\tau\ln(1+t/\tau)}$

The great pumpkin drop

depending on sign of v_y

$\dot{v}_y = -mg \pm cv_y^2$ say ball is dropped $\Rightarrow$ v_y is negative $\Rightarrow$ use $+$ sign

$\dfrac{dv_y}{-mg+cv_y^2} = dt$ Integrate both sides again $-\dfrac{\tanh^{-1}\left(\sqrt{\frac{c}{mg}}\,v_y\right)}{\sqrt{mgc}} = t - t_0$ $\quad \boxed{v_y = -\sqrt{\frac{mg}{c}}\tanh\left(\sqrt{mgc}\,t\right)}$ $\quad v_{terminal} = -\sqrt{\frac{mg}{c}}$

asymptotes to 1

Aside:
So what is c? Depends on x-sec area & properties of medium

Since full eq. is coupled & non-linear, solutions do not obey superposition principle (sum is not a sol'n)

can't put together x & y solutions → not even analytic (need computer algorithm)

Coupled Linear Equations

Charged particle in uniform magnetic field

$\vec{F} = m\ddot{\vec{r}} = q\vec{v}\times\vec{B} = -v_x B\hat{y} + v_y B\hat{x} \rightarrow \dot{v}_x = v_y \frac{B}{m} \quad \dot{v}_y = -v_x \frac{B}{m} \quad \dot{v}_z = 0$

take it in $\hat{z}$ dir.

Can solve this using Complex plane: $\overset{\mathbb{C}\#}{v} = v_x + i v_y$ $\quad \dot{v} = \dot{v}_x + i\dot{v}_y = v_y \frac{B}{m} - i v_x \frac{B}{m} = -i\frac{B}{m}(v_x + i v_y) = -i\frac{B}{m}v$

So $\dot{v} = -i\frac{B}{m}v$ & $\boxed{v = v_0 e^{-i\omega t} \quad \omega/ \ \omega = \frac{B}{m}}$

Recall $e^{i\theta} = \cos\theta + i\sin\theta$

$v = (v_x^o + i v_y^o)(\cos\omega t - i\sin\omega t) = \underbrace{(v_x^o \cos\omega t + v_y^o \sin\omega t)}_{\hat{x}\text{ comp.}} + i\underbrace{(v_y^o \cos\omega t - v_x^o \sin\omega t)}_{\hat{y}\text{ component}}$

$v^o e^{i\phi} e^{-i\omega t} = v^o e^{-i(\omega t - \phi)} = v^o[\cos(\omega t - \phi) - i\sin(\omega t - \phi)]$

describes circle in complex plane!

$$\boxed{z = z_0 + \frac{v_0}{-i\omega} e^{i\phi} e^{-i\omega t} = z_0 + i\frac{v_0}{\omega} e^{-i(\omega t - \phi)}}$$

both complex