Lecture 01.1 - Physics 523

$\boxed{\text{Lecture 1}}$ Newton Said 1) Law of Inertia
2) $\vec{F} = m\vec{a}$
3) For every action, equal/opposite reaction

And we're done! Why are we here?

Applying these laws in many systems is intractable - Need more powerful formalism

1700's: Lagrangian Mechanics
1800's: Hamiltonian Mechanics
Each has their strength's & weaknesses

I. Vectors orthonormal basis vectors $\hat{e}_1, \hat{e}_2, \hat{e}_3$ (same as $\hat{x}, \hat{y}, \hat{z}$) $\vec{r} = \sum_{i=1}^{3} r_i \hat{e}_i$ $\vec{a}\cdot\vec{b} = \sum_{i=1}^{3} a_i b_i$ $\hat{a}\times\hat{b} = \sum_{ijk=1}^{3} \varepsilon^{ijk} \hat{e}_i a_j b_k$

totally anti-symmetric $\varepsilon_{123} = 1 = \varepsilon_{231} = \varepsilon_{312}$
$\varepsilon_{321} = \varepsilon_{213} = \varepsilon_{132} = -1$

derivatives $\Rightarrow$ $\frac{d\vec{r}(t)}{dt} = \lim_{\Delta t\to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \sum_{i=1}^{3} \frac{dr_i(t)}{dt}\hat{e}_i$ (non-moving basis vectors)

Time Plays a central role in physics $\rightarrow$ for us, all observers agree on common time (up to def. of t_0)

Reference Frame a couple on a rollercoaster sees physics in a much more complicated way than an observer on the ground!

Inertial Frames $\rightarrow$ basic laws e.g. Newton's manifest in standard way

In non-inertial frames, Newton's 1st + 2nd do not hold

Diff. Eq's Physics is the process of figuring out which differential equations are worth solving

$\vec{F} = m\vec{a} \longrightarrow \vec{F} = m\ddot{\vec{r}}$ Can be very easy (e.g. $x(t) = x_0 + v_0 t + \frac{F_0}{2m}t^2$) or very hard $\ddot{\phi} = \nabla^2\phi - m^2\phi + \lambda\phi^3$ non-linear

Newton's 2nd Law

$\vec{F} = m\vec{a}$ relates action ($\vec{F}$) to motion ($\vec{a}$)   Not just a conversion factor!

↑ point mass

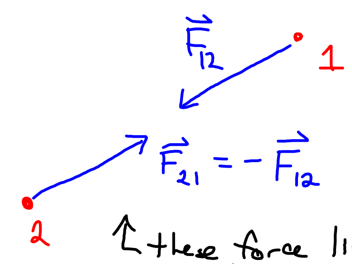
now, since momentum $\vec{p} = m\vec{v}$, and $\frac{d\vec{v}}{dt} = \vec{a}$, $\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$

$$\boxed{\vec{F} = \dot{\vec{p}}}$$

← This is the one that works in relativity

Conservation of momentum

Newton's 3rd tells us $\vec{F}_{12} = -\vec{F}_{21}$



+ any external forces $\vec{F}_1^{ext}$; $\vec{F}_2^{ext}$

↑ these force lines happen to be "central" or along line connecting particles $\vec{B}$-forces are counterexample

$$\dot{\vec{p}}_1 = \vec{F}_{12} + \vec{F}_1^{ext}$$
$$\dot{\vec{p}}_2 = \vec{F}_{21} + \vec{F}_2^{ext}$$

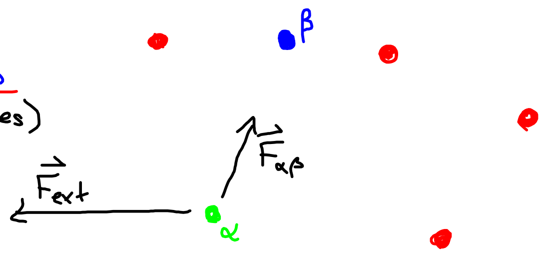
$$\dot{\vec{P}} = \dot{\vec{p}}_1 + \dot{\vec{p}}_2 = \underbrace{\vec{F}_{12} + \vec{F}_{21}}_{0 \text{ since } \vec{F}_{12} = -\vec{F}_{21}} + \vec{F}_1^{ext} + \vec{F}_2^{ext}$$

★ If external forces on any system vanish, $\boxed{\dot{\vec{P}} = \emptyset}$ $\vec{P}$ is constant

(momentum is conserved)

Multiparticle Systems

(collection of point particles)



$$\vec{F}_\alpha = \vec{F}_\alpha^{ext} + \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} = \dot{\vec{p}}_\alpha$$

total momentum $\vec{P} = \sum_\alpha p_\alpha$

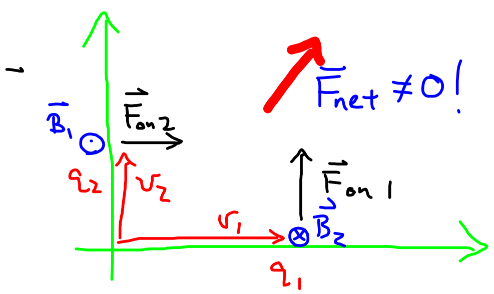
$$\dot{\vec{P}} = \sum_\alpha \dot{\vec{p}}_\alpha = \sum_\alpha \left[\vec{F}_\alpha^{ext} + \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta}\right]$$

$\frac{1}{2}\sum_{\substack{\alpha\beta \\ \alpha\neq\beta}} (F_{\alpha\beta} + F_{\beta\alpha}) \rightarrow \emptyset$ by Newton's 3rd!

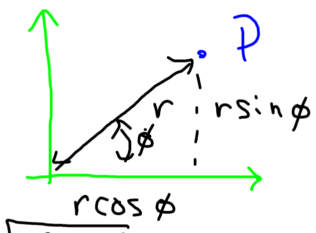
So $\dot{\vec{P}} = \sum_\alpha \vec{F}_\alpha^{ext} + \sum_{\substack{\alpha,\beta \\ (\alpha\neq\beta)}} F_{\alpha\beta}$

$$\boxed{\dot{\vec{P}} = \sum_\alpha \vec{F}_\alpha^{ext}}$$

★ Note B-forces seem to violate Newton's 3rd → patched up by accounting for momentum carried by EM waves given off by accelerating charges



Polar coordinates

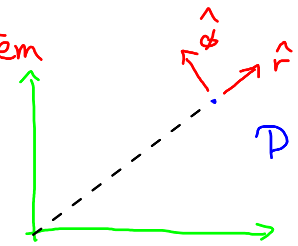


$$x = r\cos\phi \qquad y = r\sin\phi \quad \Longleftrightarrow \quad r = \sqrt{x^2+y^2} \qquad \phi = \tan^{-1}(y/x)$$

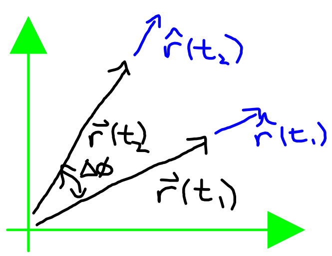
This is a "moving" coordinate system

New unit vectors $\hat{r}$ & $\hat{\phi}$:

As P changes, so do orientations of $\hat{r}, \hat{\phi}$



Consider a trajectory $\vec{r}(t)$ in polar coordinates



for small Δt, have $\hat{r}(t_2) = \hat{r}(t_1) + \hat{\phi}\,\dot{\phi}\,\Delta t$

or $\frac{d\hat{r}}{dt} = \hat{\phi}\,\dot{\phi}$

So $\dot{\vec{r}} = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$

$\ddot{\vec{r}} = \ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + (\dot{r}\dot{\phi} + r\ddot{\phi})\hat{\phi} + (r\dot{\phi})\frac{d\hat{\phi}}{dt}$

$= \ddot{r}\hat{r} + (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{\phi} - r(\dot{\phi})^2\hat{r}$

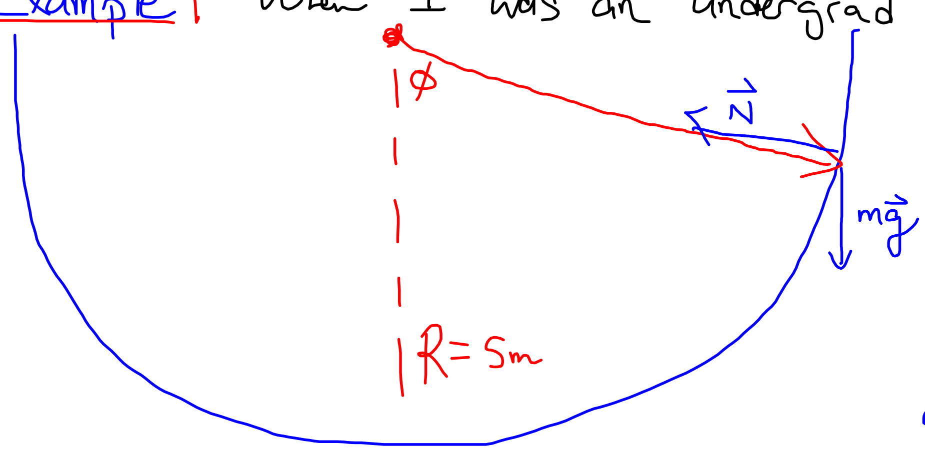
$\ddot{\vec{r}} = \hat{r}(\ddot{r} - r\dot{\phi}^2) + (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{\phi} \overset{(\dot{r}=0,\,\ddot{\phi}=0)}{=} -r(\dot{\phi})^2\hat{r} = -\frac{v^2}{r}\hat{r}$ (usual centripetal $\vec{a}$)

angular $\vec{a}$

for just $\dot{r}=0$, $\vec{a} = -r\dot{\phi}^2\hat{r} + r\ddot{\phi}\hat{\phi}$

Newton's 2nd Law: $\vec{F} = m\vec{a} \Rightarrow \begin{cases} F_r = m(\ddot{r} - r\dot{\phi}^2) \\ F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \end{cases}$

Example When I was an undergrad at Caltech, we made a half-pipe skateboard ramp



★ $r(t) = R$ (constant)

$F_r = -mR(\dot{\phi})^2 = mg\cos\phi - N$

$F_\phi = mR\ddot{\phi} \quad = -mg\sin\phi$

also a function of t!

$\ddot{\phi} = -g/r\,\sin\phi$ and $(\dot{\phi})^2 = \frac{N}{mR} - \frac{g}{R}\cos\phi$

focus on this one

$\phi > 0 \Rightarrow \ddot{\phi} < 0$ (accelerates to the left) $\rightarrow$ reverses for $\phi < 0$ OSCILLATES

But Eq. is too complicated (sol'n is a Jacobi Elliptic Function)

appear in Jacobi Elliptic's too!

tbd by boundary conditions

for small ϕ, $\sin\phi \approx \phi$, and eq. becomes linear $\ddot{\phi} = -\frac{g}{R}\phi$

$$\boxed{\phi(t) = A\cos\omega t + B\sin\omega t \qquad \omega \equiv \sqrt{\frac{g}{R}}}$$

Just a SHO (like everything)

Initial Conditions: $\phi(t=0) = 0 \Rightarrow \phi(t) = B\sin\omega t$ $(A=0)$

$\dot{\phi}(t=0) = \omega B\cos(0) = \omega B$ $\left(B = \frac{\dot{\phi}_0}{\omega}\right)$

$$\boxed{\phi(t) = \frac{(\dot{\phi}_0)}{\omega}\sin\omega t}$$

Drag forces Air resistance, viscous drag, lift

force $\perp$ to motion

falling objects + terminal velocity

In many cases F_{drag} is in opposite direction of velocity: $f(v) = -f(v)\hat{v}$

can be very complicated (turbulence, etc.)

unit vector in dir. of $\vec{v}$

viscous drag (jelly-like resistance)

Taylor expand $f(v) = bv + cv^2 + \mathcal{O}(v^3)$

air type resistance

for air can usually neglect b, for jelly, can usually neglect c (See example 2.1)

Linear drag: $F = m\dot{v} = -bv$ $\quad \dot{v} = -\frac{b}{m}v \Rightarrow \boxed{v = v_0 e^{-\frac{b}{m}t}}$ Motion slows exponentially