$\boxed{\text{HW\# 10 Solutions}}$

① 8-2 In French Recall $\dfrac{dKE}{dx} = \dfrac{1}{2}\mu\left[f'(t \mp x/v)\right]^2$

for a single traveling wave, and $\dfrac{dKE}{dx} = \dfrac{dPE}{dx}$

So $E_{tot} = \mu\left[f'(t \mp x/v)\right]^2$.

→ We have 3 waves:
1) incident $f_1(t - x/v_1)$
2) reflected $g_1(t + x/v_1)$
3) transmitted $f_2(t - x/v_2)$

Want to show $E[f_1] = E[g_1] + E[f_2]$

$$E[f_1] = \mu_1 \int dx \left[f_1'(t - x/v_1)\right]^2$$

$E[g_1] = \mu_1 \int dx \left[g_1'(t + x/v_1)\right]^2$ and $g_1(t) = \dfrac{v_2 - v_1}{v_2 + v_1} f_1(t)$

and thus $g_1(t + x/v_1) = \dfrac{v_2 - v_1}{v_2 + v_1} f_1(t + x/v_1)$

$$\Rightarrow E[g_1] = \mu_1 \int dx \left[\frac{v_2 - v_1}{v_2 + v_1}\right]^2 \left[f_1'(t + x/v_1)\right]^2$$

$(x \leftrightarrow -x)$

$$= \mu_1 \left(\frac{v_2 - v_1}{v_2 + v_1}\right)^2 \int dx \left[f_1'(t - x/v_1)\right]^2$$

$$= \left(\frac{v_2 - v_1}{v_2 + v_1}\right)^2 E[f_1]$$

$E[f_2] = \mu_2 \int dx \left[f_2'(t - x/v_2)\right]^2$ and $f_2(t) = \dfrac{2v_2}{v_2 + v_1} f_1(t)$

so $f_2(t - x/v_2) = \dfrac{2v_2}{v_2 + v_1} f_1(t - x/v_2)$

$$\Rightarrow E[f_2] = \mu_2 \left(\frac{2v_2}{v_2 + v_1}\right)^2 \int dx \left[f_1'(t - x/v_2)\right]^2$$

take $x = x'(v_2/v_1) \Rightarrow dx = dx'(v_2/v_1)$

$$E[f_2] = \mu_1 \underbrace{\left(\frac{\mu_2}{\mu_1}\right)}_{(v_1/v_2)^2} \int dx' \left(\frac{v_2}{v_1}\right) \left[f_1'(t - x'/v_1)\right]^2 \left(\frac{2v_2}{v_2+v_1}\right)^2$$

$$= \mu_1 \left(\frac{4v_1v_2}{(v_2+v_1)^2}\right) \int dx' \left[f_1'(t - x'/v_1)\right]^2$$

$$= \left(\frac{4v_1v_2}{(v_2+v_1)^2}\right) E[f_1]$$

So $E[g_1] + E[f_2] = \left[\left(\frac{v_2 - v_1}{v_2+v_1}\right)^2 + \frac{4v_1v_2}{(v_2+v_1)^2}\right] E[f_1]$

$$= 1 \times E[f_1]$$

* Thus, energy flux is conserved at the junction

② 8-5 in French The boundary conditions for the waves impinging upon the boundary between air and water are different relative to the waves on a string.

→ Continuity of displacement: $\xi_1(x=0,t) = \xi_2(x=0,t)$

→ Continuity of pressure: $K_1 \frac{\partial \xi_1}{\partial x}(x=0,t) = K_2 \frac{\partial \xi_2}{\partial x}(x=0,t)$

a) Following the style of the derivation for waves on a string: $\xi_1(x,t) = f_1(t - x/v_1) + g_1(t + x/v_1)$

$$\xi_2(x,t) = f_2(t - x/v_2)$$

The B.C.'s at the junction require: $f_1(t) + g_1(t) = f_2(t)$

and $\frac{K_1}{v_1}(f_1(t) - g_1(t)) = \frac{K_2}{v_2} f_2(t)$

Solving for g_1, f_2 in terms of f_1 @ the junction:

$$g_1(t) = \left(\frac{v_2/K_2 - v_1/K_1}{v_2/K_2 + v_1/K_1}\right) f_1(t) \; ; \; f_2(t) = \frac{2\, v_2/K_2}{(v_2/K_2 + v_1/K_1)} f_1(t)$$

→ generalizing to arbitrary points on the string:

$$g_1(t + x/v_1) = \left(\frac{v_2/K_2 - v_1/K_1}{v_2/K_2 + v_1/K_1}\right) f_1(t + x/v_1)$$

$$f_2(t - x/v_2) = \left(\frac{2\, v_2/K_2}{v_2/K_2 + v_1/K_1}\right) f_1(t - x/v_2)$$

→ The second of these relations gives the ratio of transmitted to incident waves as:

$$\frac{A_t}{A_i} = \frac{2\, v_2/K_2}{v_2/K_2 + v_1/K_1} = \frac{2 v_2}{v_2 + v_1 (K_2/K_1)}$$

and $K_2/K_1 = \left(\frac{v_2}{v_1}\right)^2 \frac{\rho_2}{\rho_1}$ and $\rho_{air} = 1.2\,kg/m^3$

$\rho_{water} = 1000\,kg/m^3$

thus $\boxed{\frac{A_t}{A_i} = 5.4 \times 10^{-4}}$

~~b) Since energy flux scales as~~ $\cancel{\frac{A^2}{v}}$, ~~we have~~

$$\cancel{\frac{E_{trans}}{E_{incident}} = \frac{A_t^2/v_{water}}{A_i^2/v_{air}} = \left(\frac{A_t}{A_i}\right)^2 \frac{v_{air}}{v_{water}} =}$$

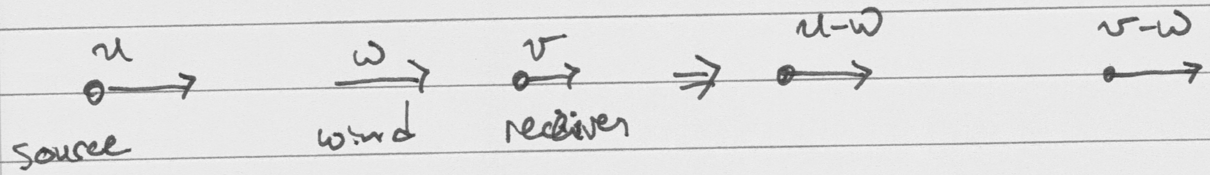
b) Following #1 in HW, but with

$$\frac{dKE}{dx} = \frac{1}{2} \rho \underbrace{S}_{\text{surface area}} [f'(t \mp x/v)]^2, \text{ and } f_2 \sim \frac{2\, v_2/K_2}{v_2/K_2 + v_1/K_2} f_1,$$

then
$$E[f_2] = E[f_1] \times \frac{4 v_2^3/v_2 / K_2^2 \cdot \frac{\rho_2}{\rho_1}}{(v_2/K_2 + v_1/K_1)^2} = \frac{4 v_2^3/v_1 \; \rho_2/\rho_1 \; E[f_1]}{\left(v_2 + v_1 \left(\frac{v_2}{v_1}\right)^2 \frac{\rho_2}{\rho_1}\right)}$$

$$\Rightarrow \boxed{E[f_2] = 1 \times 10^{-3} E[f_1]}$$

③ 8-11 in French It is convenient in this case to go to the frame of reference where the air is still:



→ use standard formula for frequency due to moving source :

$$f_{air} = \frac{f_0}{1 - \frac{u-w}{V}} = \frac{f_0 V}{V-u+w}$$

where V is speed of sound in still air

→ Now must obtain frequency perceived by moving observer

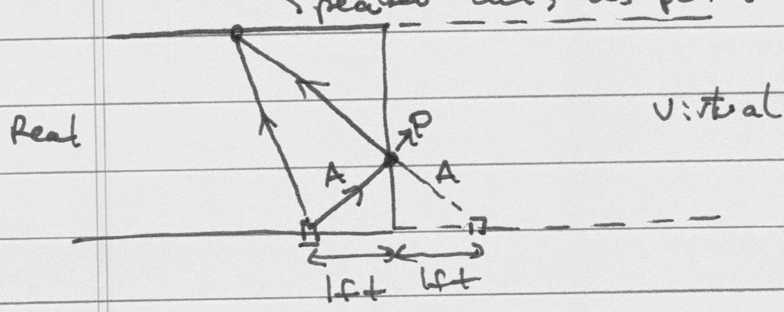
$$T_{obs} = \frac{\lambda}{V_{rel}} = \frac{\lambda}{V-(v-w)} = \frac{\lambda}{V-v+w} = \frac{V/f_{air}}{V-v+w}$$

↳ relative vel. of wave / observer

and $\boxed{f_{obs} = \frac{1}{T_{obs}} = \frac{V-v+w}{V} f_{air} = f_0 \frac{V-v+w}{V-u+w}}$

④ 8-14 in French Draw ray diagram to obtain interference pattern on far wall:

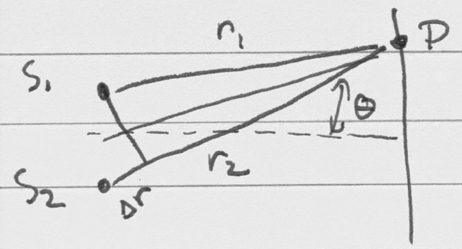
Speaker acts as point source



* to obtain correct B.C. for wave at P, require virtual speaker to create equal & opposite wave (π out of phase)

⇒ Interference caused by reflection identical to that of 2 point sources π out of phase.

Rederive interference pattern:



$$\xi_p = \xi_1 \cos\left(\frac{2\pi}{\lambda}(r_1 - vt)\right) + \xi_2 \cos\left(\frac{2\pi}{\lambda}(r_2 - vt)\right)$$

★ Note, since sources are π out of phase, $\xi_2 = -\xi_1$

$$\xi_p = \xi_0 \left[\cos\left(\frac{2\pi}{\lambda}(r_1 - vt)\right) - \cos\left(\frac{2\pi}{\lambda}(r_2 - vt)\right)\right]$$

$$= +2\xi_0 \sin\left[\frac{2\pi}{\lambda}(\bar{r} - vt)\right] \sin\left[\frac{\pi}{\lambda}(r_2 - r_1)\right] \quad \text{and } \delta r \simeq d\sin\theta$$

$$\Rightarrow A(\theta) = 2\xi_0 \sin\left[\frac{\pi}{\lambda} d\sin\theta\right]$$

★ Note interference is dest. at $\theta = 0$ (must be, as wall enforces fixed B.C.!

→ Pattern is similar to two in-phase sources, but bright fringes exchanged for dark fringes.

b) for a 12'x18' room, $\theta_{max} = \tan^{-1}(18/12) \simeq 1$ radian

to fit 2 fringes, need $\frac{\pi}{\lambda} d\sin\theta_{max} \gtrsim 2\pi$

$$\Rightarrow f \gtrsim 2v / d\sin\theta_{max} = 2(334\,\text{m/s}) \Big/ \left[(2\,\text{feet})\left(\frac{\text{meter}}{3.3\,\text{ft}}\right)(.84)\right]$$

$$\Rightarrow \boxed{f \gtrsim 1312\,\text{Hz}}$$

To get destructive interference at 3', or

$\theta_3 = \tan^{-1}\left(\frac{3}{12}\right) = .245$, need $\frac{f}{v}\pi d\sin\theta_3 = n\pi$

$$\Rightarrow \boxed{f_n = \frac{nv}{d\sin\theta_3} = n(2272\,\text{Hz})}$$

So integer multiples of $f \sim 2300\,\text{Hz}$ are suppressed