HW#8 Solutions

① 7-2 In French

We are given a wave-form $y(x,t) = (0.3)\sin(\pi(0.5x - 50t))$
with $[y] = [x] = cm$, and $[t] = s$.

a) Amplitude of wave is $A = 0.3cm$

Wavelength: $\frac{2\pi}{\lambda} = \frac{0.5\pi}{cm} \Rightarrow \lambda = 4cm$

Wavenumber $k = \frac{1}{\lambda} = 0.25\, cm^{-1}$

Frequency $\omega = 50\pi = 2\pi f \Rightarrow f = 25\, s^{-1}$

Period $T = \frac{1}{f} = 0.04s$

velocity $v = \lambda f = (4cm)(25s^{-1}) = 100\, cm/s$

b) $\frac{\partial y}{\partial t} = (0.3)(-\pi \cdot 50)\cos(\pi(0.5x - 50t))$

$\Rightarrow v_{\perp}^{max} = 50\pi(0.3)\, cm/s = 524\, cm/s$

② 7-5 in French Long uniform string, $\mu = 0.1 kg/m$
$T = 50N$

$A = .02m$, $T = 0.1s$

a) $v_{wave} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50N}{0.1kg/m}} = 22.4 m/s$

b) $\lambda = \frac{v}{f} = Tv = (0.1s)(22.4 m/s) = 2.2m$

c) The general form of the wave is

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt) + \delta\right]$$

↑ phase we need to solve for

$$= (0.02\text{m})\sin\left[\frac{2\pi}{(2.2\text{m})}(x - 22.4\,\text{m/s}\,t) + \delta\right]$$

given that $y(x=0, t=0) = (0.02\,\text{m})\sin\delta = 0.01\,\text{m}$

and $\left.\frac{\partial y}{\partial t}\right|_{x=0,\ t=0} < 0$

$\Rightarrow$ $\boxed{\delta = \pi/6 \text{ radians}}$ (Note velocity info tells us $\delta \neq \frac{5\pi}{6}$)

So $\boxed{y(x,t) = (0.02\text{m})\sin\left[\frac{2\pi}{(2.2\text{m})}(x - (22.4\,\text{m/s})t) + \frac{\pi}{6}\right]}$

(3) 7-8 in French For a single traveling wave, we have

$y(x_1, t) = 0.2\sin(3\pi t)$ $(x_1 = 0\text{m})$

$y(x_2, t) = 0.2\sin(3\pi t + \pi/8)$ $(x_2 = 1\text{m})$

a) $\omega = 3\pi = 2\pi f \Rightarrow \boxed{f = \frac{3}{2}\,s^{-1}}$

b) We have many possible answers depending on ~~direction wave is traveling, and~~ on how many complete cycles the wave goes through over 1m.

we have $\pm \frac{2\pi}{\lambda}(1\text{m}) + n(2\pi) = \frac{\pi}{8}$

$\Rightarrow$ $\boxed{\lambda = \pm\left(\frac{1}{16} + n\right)^{-1}\text{m}}$ with n unknown

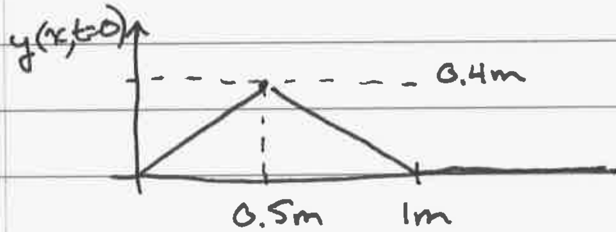
Note: when λ is negative, means wave is right moving. positive $\Rightarrow$ left M.

c) $v = f\lambda = \left(\frac{3}{2} s^{-1}\right)\left(\frac{1\,m}{\frac{1}{16}+n}\right) = \frac{24\ m/s}{1+8n}$

d) As seen in part b, it is unclear whether the wave is L.M. or R.M.

④ Problem 7-9 in French

at $t=0$, the pulse looks like:



and has form $y(x,t) = f(x-vt)$ for all other times

we can relate the transverse velocity at $x=1m$ to the shape at $t=0$ by noting $\frac{\partial y}{\partial t} = -v\frac{\partial f}{\partial x}$

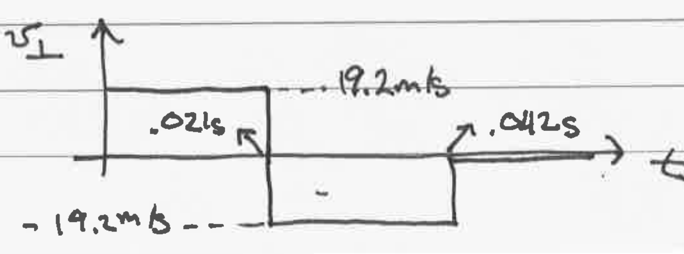
So $\left.\frac{\partial y}{\partial t}\right|_{x=1m} = -v\left.\frac{\partial f}{\partial x}\right|_{x=1m}$

As the wave passes by $x=1m$, at first the slope is negative, and given by $\frac{\partial f}{\partial x} = \frac{-0.4m}{0.5m} = -0.8$

$\Rightarrow \frac{\partial y}{\partial t} = (-24 m/s)(-0.8) = 19.2 m/s$ ← given wave vel.

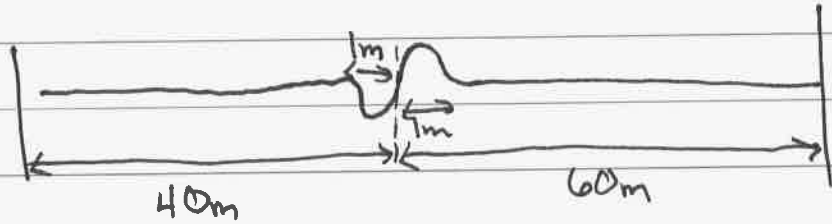
at time $t = \frac{0.5m}{24 m/s} = .021s$, the transverse velocity reverses,

until $t = .042s$, when it drops to zero



(5) Problem 7-12 in French

a)



b) The transverse velocity for a right-moving wave is $\frac{\partial y}{\partial t} = -v_{wave} f'(x-vt)$

The slope $|f'|$ is about $\left|\frac{0.1m}{1m}\right| \simeq .1$

$$\Rightarrow \boxed{\left|\frac{\partial y}{\partial t}\Big|_{max}\right| \simeq \left|(-40m/s)(.1)\right| = 4m/s}$$

c) $M_{string} = 2kg \Rightarrow \mu = \frac{2kg}{100m} = .02 kg/m$

↳ length of string

$v_{wave} = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v_{wave}^2 = (.02kg/m)(40m/s)^2 = 32N$

$$\boxed{T \simeq 32N}$$

d) $y(x,t) = f(x+vt)$ for left-moving

$= A\sin\left(\frac{2\pi}{\lambda}(x+vt)\right)$ for sinusoidal waves

$$\boxed{y(x,t) = (0.2m)\sin\left[\frac{2\pi}{(5m)}\left(x+\left(\frac{40m}{s}\right)t\right)\right]}$$