Reading and HW #3 PHY360 Fall 2016 Due Friday 9/23/2016

Reading

Please read Chapters 3 and 4 of the textbook (A.P. French, "Vibrations and Waves").

Problem #1
Rod Pendulum

Consider a pendulum made by nailing a rod of total length L to a wall, with the nail put through the rod at a point $1/3L$ from the top of the rod. Considering small oscillations only, what is the period of small oscillations of the rod if the system is perturbed? Hint: Calculate the moment of inertia of the rod when it rotates about the point where the nail is, and then refer to pages 51-53 in the text.

Problem #2
Hoop Pendulum

Consider a pendulum made by hanging a hoop with diameter d on a nail. What is the period of oscillations at small amplitude?

Problem #3
Decaying oscillations

The motion of a 1 dimensional oscillator can be represented by a graph in which the position $x(t)$ is shown on the horizontal axis, while the vertical axis indicates the instantaneous velocity $\dot{x}$.

(a) Show that for an *undamped* oscillator that this curve is an ellipse using an important conservation law.

(b) Show qualitatively (or create a plot using a program like mathematica - printing and submitting all code you use to generate it) that the addition of a damping term makes the graph a curve that spirals inwards towards the origin as t increases.

Problem #4
Decaying oscillations

Verify that $x(t) = Ae^{-\alpha t}\cos\omega t$ is a possible solution of the equation

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \tag{1}$$

Find α and ω in terms of γ and ω_0. You may use trig, if you like, or go to complex exponential notation (which is simpler, in my view).

Problem #5
Quality Factor

The *quality factor* associated with a damped harmonic oscillator is given by $Q = \omega_0/\gamma = \frac{\sqrt{k/m}}{(b/m)}$. Show that this is a dimensionless number. You will need to work out the units of b, which you can do by comparison of the 3 terms in the linear homogenous equation governing the evolution of a damped harmonic oscillator. Describe qualitatively what it means to have a damped oscillator with a high Q value vs a low Q value.