Reading and HW #2 PHY360 Fall 2016 Due Fri 9/16/2016

Reading Please read Chapters 2 and 3 of the textbook (A.P. French, "Vibrations and Waves").

Problem #1
Superposition!

You are given the following superpositions of oscillations with equal frequency, but varying amplitude and phase. Express these various sums as a single complex exponential $z(t)$, such that $x(t) = \text{Re}[z(t)]$:

(a) $x(t) = \cos\omega t + \sin(\omega t + \pi/4)$

(b) $x(t) = \cos(\omega t + \pi/3) + \cos\omega t$

(c) $x(t) = 3\sin\omega t + 2\cos\omega t$

(d) $x(t) = \sin\omega t + \cos\omega t - 2\cos(\omega t + \pi/4)$

This exercise is repetitive, but hopefully will make you feel very comfortable with the summation of various complex exponentials to obtain the resulting superposition of many pure sinusoidal oscillations. Speaking as a runner, sometimes it pays to just put a few miles on the shoes.

Problem #2
Periodicity of Superposed Oscillations

Consider two oscillations that occur in a system, both along the same direction, but with different frequencies that are multiples of some given frequency, ω_0. Give the period of the superposition of the two waves in the following instances:

(a) $x_1(t) = A_1\cos(\omega t)$ and $x_2(t) = A_2\cos(2\omega t)$

(b) $x_1(t) = A_1\cos(2\omega t)$ and $x_2(t) = A_2\cos(3\omega t)$

(c) $x_1(t) = A_1\cos(n\omega t)$ and $x_2(t) = A_2\cos(m\omega t)$ where n and m are both integers.

(d) $x_1(t) = A_1\cos(\omega t)$ and $x_2(t) = A_2\cos(\pi\omega t)$

Note that it is possible for there to be no periodicity of the resulting sum!!!

Problem #3
Beat Phenomenon

Show that the superposition of two waves of two different frequencies, but identical amplitudes (i.e. $x_1(t) = A\cos\omega_1 t$, and $x_2(t) = A\cos(\omega_2 t)$ can be expressed as the following formula:

$$x(t) = x_1(t) + x_2(t) = 2A\cos\left[\frac{1}{2}\Delta\omega t\right]\cos\left[\bar{\omega}t\right] \tag{1}$$

where $\Delta\omega = \omega_2 - \omega_1$, and $\bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2)$. The use of complex exponentials to get this result is strongly encouraged. As another hint, I encourage you to replace ω_1 and ω_2 with $\Delta\omega$ and $\bar{\omega}$ early on in the calculation. Be sure to speak with your classmates, Raghav, or me for help if you need it.

Problem #4
Double Springs

Compute the following problems regarding a mass suspended from above by one or more springs.

(a) Consider a single mass m suspended from the ceiling by a spring with spring constant k. What is the period of oscillation of the mass if it is displaced from equilibrium and let go?

(b) Consider now adding a second identical spring to the system one end attached to the mass, the other to the ceiling, just like the first one. Now that there are two springs with constant k attaching the mass to the ceiling, what is the new period of oscillation of the system if it is displaced from equilibrium?

(c) Consider now attaching the same mass to the same two springs, but add the springs such that one is attached to the ceiling, the next is attached to the bottom of the first, and the mass finally attached to the end of the two-spring-long system. Now what is the period of oscillation?

Give all results in terms of k and m.