Reading and HW #1 PHY360 Fall 2016 Due 9/9/2016 5pm

Reading

Please read Chapters 1 and 2 of the textbook (A.P. French, "Vibrations and Waves").

Problem #1
Mass+Spring

Consider a spring, itself massless, hanging from the ceiling. The spring is 20cm long. Now hang a mass M from the bottom of the spring. You at first support it with your hand, leaving the spring relaxed, and then you suddenly remove the supporting hand. The mass and the spring oscillate, and you measure the lowest position of the mass during it's motion to be 10cm below the initial place you were holding it.

(a) What is the frequency of oscillation?

(b) What is the velocity when the mass is 5cm below its initial resting place?

(c) A second mass of 300gm is attached to the first, so that the total mass is now $M_{\text{new}} = M + 300\text{gm}$. You note that when this system oscillates, it does so with half the frequency of the system with just mass M. What is M?

(d) With that new mass, where is the new equilibrium position of the mass M_{new}?

Problem #2
Complex Exponentials

Euler's relation, $e^{i\theta} = \cos\theta + i\sin\theta$ is fantastically useful, and so I would like you to gain some familiarity with manipulations using complex exponentials.

(a) Express $e^{-i\theta}$ purely in terms of $\cos\theta$ and $\sin\theta$.

(b) Express $\cos\theta$ purely in terms of a sum/difference of complex exponentials

(c) Express $\sin\theta$ purely in terms of a sum/difference of complex exponentials

(d) Prove the trig identity $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$ using complex exponentials

(e) Prove the trig identity $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$ using complex exponentials

Problem #3
Derivatives of C.E.'s

Show explicitly that taking the derivative with respect to θ of both sides of the equation

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{1}$$

Yields the same equation as multiplying both sides by i.

Problem #4
Taylor Series and C.E.'s

Taylor expand both sides of the following two equations, and show that the terms match order-by-order in the expansion:

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right) \tag{2}$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) \tag{3}$$

Problem #5
Sinusoids and S.H.O.'s

Show that $y(t) = A\cos\omega t + B\sin\omega t$ is a solution to the equation $\ddot{y}(t) = -\omega^2 y(t)$, where A and B are arbitrary constants. Now use complex exponential notation, and show that this solution can also be written as $y(t) = C\cos(\omega t + \alpha) = C\text{Re}[e^{i(\omega t+\alpha)}] = \text{Re}[(Ce^{i\alpha})e^{i\omega t}]$.

Problem #6
Circular motion and S.H.O.'s

A point moves in a circle at constant speed of 100 cm/s. The period of a complete cycle of rotation is 6 seconds. At $t = 0$, the line to the point from the center of the circle makes an angle of $\pi/6$ radians with respect to the x-axis.

(a) Find the equation of the x-coordinate of the point as a function of time, in the form $x = A\cos(\omega t + \alpha)$, and give the numerical values of A, ω, and α.

(b) Find the values of $x(t)$, $\dot{x}(t)$, and $\ddot{x}(t)$ at $t = 2$ seconds.

Problem #7
Mathematica and Oscillation

Install Mathematica or find a workstation that has it and play with the code that is available on the website for week 1.