Lecture 2.1

Most of this discussion is tied closely to what is going on in Chapter 2 of A.P. French's Book

Subject: The Superposition Principle

Almost all vibrations have company, 1) Road vibration 2) Engine Vibration 3) Heart stopping bass } How do we interpret this cacaphony of oscillation?

★ Key point: Many systems have linear response. What does this mean?

Mass on a spring is "linear". Say you have a system w/ natural oscillation frequency ω: $\ddot{x} = -\omega^2 x$

Look at two solutions: $x_1(t) = A_1\cos(\omega t + \alpha_1)$, $\quad \ddot{x}_1 = -\omega^2 x_1$
$x_2(t) = A_2\cos(\omega t + \alpha_2)$, $\quad \ddot{x}_2 = -\omega^2 x_2$
$\Big\}\ \ddot{x}_1 + \ddot{x}_2 = -\omega^2 x_1 - \omega^2 x_2$

$\Rightarrow (\ddot{x_1 + x_2}) = -\omega^2(x_1 + x_2)$ Sum is also solution!

Show it doesn't work for non-linear! (HW)

Another lesson: The "mass-on-a-spring" is UBIQUITOUS therefore so is the Superposition Principle: Effect of 2 or more vibrations is sum of individual vibrations

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How shall we deal with this mathematically? Our geometric picture will be of aid → complex #s sum like vectors

I. Summing Waves of Equal frequency

Exercise at board → Have 2 sinusoidal motions, represented by complex #'s via

$x_1(t) = \mathrm{Re}[z_1(t)] \qquad z_1(t) = A_1 e^{i(\omega t + \alpha_1)}$

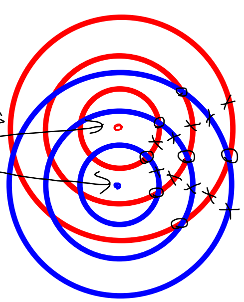
$x_2(t) = \mathrm{Re}[z_2(t)] \qquad z_2(t) = A_2 e^{i(\omega t + \alpha_2)}$

Students draw both z_1, z_2 + sum at time $t=0$ (assume both vectors start in same quadrant, 1st)

What do z_1, z_2, and sum look like at time $t = \frac{\pi}{\omega}$?

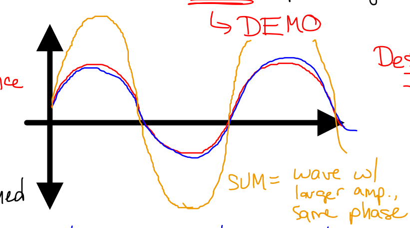
Consider 2 generators of pure sinusoidal waves, i.e. sound or water, precisely at same phase, but spaced some distance apart

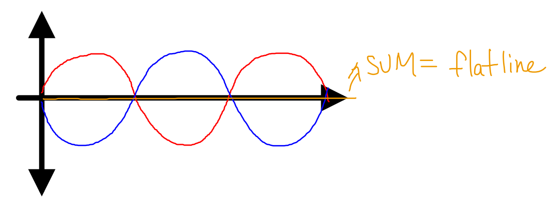
↳ DEMO



Amplitudes
O = Add → complex #s aligned

X = Subtract → complex #s anti-aligned





If these are sound waves, sound is LOUD at O, and soft at X

As you move, you experience either CONSTRUCTIVE interference, or DESTRUCTIVE interference

LESSON → phenomena of superposed waves very rich

Now imagine we have 2 oscillators generating waves, but at different frequencies

I. Beat Phenomenon The pattern of positive and negative interference in the previous example is static. If you sit at an x, it will stay "quiet"

And at 0, stays "loud"

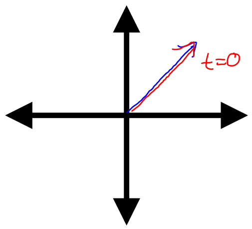
*One important aspect of superposition:
We saw that you can build various periodic functions from pure sinusoids (beads + straws on rods demo) with Different Frequencies

Fourier: Method of systematic expansion to characterize complicated periodic phenomena

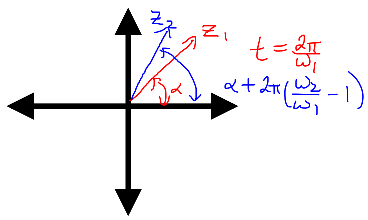
DEMO: Mistuned tuning fork – time varying amplitude, not just space-varying!

$z_1 = A\, e^{i(\omega_1 t + \alpha)}$
$z_2 = A\, e^{i(\omega_2 t + \alpha)}$ } 2 waves, same amplitude (for simplicity) same phase (initial condition)

$\omega_2 \gtrsim \omega_1$



$x = \mathrm{Re}[Ae^{i\alpha} + Ae^{i\alpha}]$
$= 2A\cos\alpha$



$x = \mathrm{Re}[z_1 + z_2]$
$= \mathrm{Re}\left[Ae^{i(2\pi+\alpha)} + Ae^{i(2\pi\frac{\omega_2}{\omega_1}+\alpha)}\right]$
$= A\left[\cos(2\pi+\alpha) + \cos\left(2\pi\left(\frac{\omega_2}{\omega_1}\right)+\alpha\right)\right]$

Generally,

$$\boxed{x = 2A\cos\Big(\underbrace{\frac{\omega_2-\omega_1}{2}}_{\frac{1}{2}\Delta\omega}t\Big)\cos\Big(\underbrace{\frac{\omega_1+\omega_2}{2}}_{\bar{\omega}\ \text{(avg. freq.)}}t\Big)}$$

HW: Prove this → use complex exponentials!

Beat frequency → amplitude has maxima at frequency $\omega_2 - \omega_1 = \Delta\omega$ (Why not $\frac{\omega_2-\omega_1}{2}$?)

Show plot of superposed waves of almost same frequency, and then very different ones