Lecture 13.1

<u>Recall:</u> Studying reflections/transmission through discontinuous change of medium:

<u>String</u>

I.	II.
μ_1	μ_2
$v_1 = \sqrt{\frac{T}{\mu_1}}$	$v_2 = \sqrt{\frac{T}{\mu_2}}$

$x=0$

wave equation in each region holds, with velocity v_1 and v_2

How to solve for traveling wave?

1. Set up solutions generally

I

$$y_1(x,t) = \underbrace{f_1\left(t - \frac{x}{v_1}\right)}_{\text{initial wave}} + \underbrace{g_1\left(t - \frac{x}{v_2}\right)}_{\text{wave reflected by discontinuity}}$$

II

$$y_2(x,t) = \underbrace{f_2\left(t - \frac{x}{v_2}\right)}_{\text{wave transmitted through discontinuity}}$$

* Conditions at junction of media:

i) $y_1(x=0,t) = y_2(x=0,t) \Rightarrow \boxed{f_1(t) + g_1(t) = f_2(t)}$

ii) $\left.\frac{\partial y_1}{\partial x}\right|_{x=0} = \left.\frac{\partial y_2}{\partial x}\right|_{x=0}$ (So acceleration is finite!)

$$\Rightarrow -\frac{1}{v_1}\left(f_1'(t) - g_1'(t)\right) = -\frac{1}{v_2} f_2'(t)$$

$$\Rightarrow f_1(t) - g_1(t) = \frac{v_1}{v_2} f_2(t)$$

<u>Then solve:</u>

$$2 f_1(t) = (1 + v_1/v_2) f_2(t) \Rightarrow \boxed{f_2(t) = \left(\frac{2v_2}{v_1+v_2}\right) f_1(t)}$$

$$2 g_1(t) = (1 - v_1/v_2) f_2(t) \Rightarrow \boxed{g_1(t) = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) f_1(t)}$$

(2)

Adding back proper x-dependence:

$$g_1(t + x/v_1) = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) f_1(t + x/v_1)$$

$$f_2(t - x/v_2) = \left(\frac{2v_2}{v_2 + v_1}\right) f_1(t - x/v_2)$$

note, arguments of f_1 modified so wave eq. is solved in the two regions

Useful check: energy must be conserved! Does $E[f_1] = E[g_1] + E[f_2]$?

$$E[f_1] = 2\int \frac{dKE}{dx}\, dx \quad \text{(recall } KE = PE \text{ so } E_{tot} = 2\, KE_{tot}\text{)}$$

$$= \int \mu_1 (f_1'(t - x/v_1))^2\, dx = \mu_1 \int dx\, [f_1'(t - x/v_1)]^2$$

$$E[g_1] = \int \mu_1 (g_1'(t + x/v_1))^2\, dx = \mu_1 \int dx \left(\frac{v_2 - v_1}{v_2 + v_1}\right)^2 [f_1'(t + x/v_1)]^2$$

change variables $x \leftrightarrow -x$ $\quad = \mu_1 \int dx \left(\frac{v_2 - v_1}{v_2 + v_1}\right)^2 [f_1'(t - x/v_1)]^2$

$$E[f_2] = \int \mu_2 [f_2'(t - x/v_2)]^2 = \int dx\, \mu_1 \left(\frac{v_1}{v_2}\right)^2 \left(\frac{2v_2}{v_2 + v_1}\right)^2 [f_1'(t - x/v_2)]^2$$

change variables $x \to v_2/v_1\, x$: $\quad \int \mu_1 \left(\frac{2v_1}{v_2 + v_1}\right)^2 \underbrace{\left(\frac{v_2}{v_1}\right)}_{dx \to v_2/v_1\, dx} [f_1'(t - x/v_1)]^2 dx$

$$= \mu_1 \int \frac{4 v_1 v_2}{(v_2 + v_1)^2} [f_1'(t - x/v_1)]^2 dx$$

$$E[g_1] + E[f_2] = \underbrace{\left[\left(\frac{v_2 - v_1}{v_2 + v_1}\right)^2 + \frac{4 v_1 v_2}{(v_2 + v_1)^2}\right]}_{1} \mu_1 \int dx\, [f_1'(t - x/v_1)]^2$$

$$= E[f_1] \;!!$$

③

Phenomena of waves in more than 1 dimension

e.g. waves on a surface, or sound in a room

→ reflection / refraction of plane waves

→ doppler effect

→ interference of point sources (i.e. double slit)

Wave equation in 2D: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 z}{\partial t^2}$

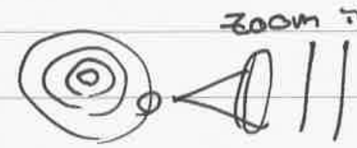
often useful in polar coordinates: $\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} = \frac{1}{v^2}\frac{\partial^2 z}{\partial t^2}$

if circular symmetric, $\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} = \frac{1}{v^2}\frac{\partial^2 z}{\partial t^2}$

Wave types of particular importance: I. Plane waves

II. Circular / Spherical waves

* all waves, sufficiently far from source, look like plane waves:



look like parallel wave fronts!

Plane waves:

<u>Demonstration</u> show plane waves in water tank

<u>Circular waves</u> Huygens principle → any wave front can be made up of simple point sources creating circular (or spherical, in 3D) wave fronts

won't go in more depth than this... only useful as pictorial guide

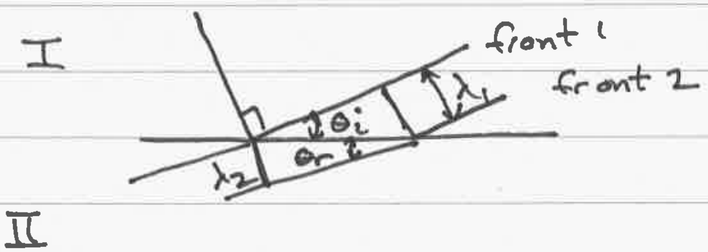
④

Reflection / Refraction of plane waves

→ the 2D analog of the two strings, mass/length μ_1/μ_2

→ presume plane waves impinging on boundary between 2 media: wave speed changes, direction may also change

Can derive Snell's law by geometry:



* frequency must be same on both sides

$\lambda_1 f = v_1$

$\lambda_2 f = v_2$

now $\sin\theta_i = \dfrac{\lambda_1}{h}$ ← hypotenuse $\qquad \sin\theta_r = \dfrac{\lambda_2}{h}$

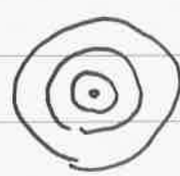
$$\Rightarrow \boxed{\frac{\sin\theta_i}{\sin\theta_r} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}}$$

→ there is also a reflected wave as well as before signal → reflected + transmitted

⑤

Doppler Effect

→ When a wave source moves with respect to the medium, wave pattern is affected

Stationary: 

moving:

$$\lambda_{right} = \lambda_0 - v_s T = \lambda_0 - \frac{v_s}{f} = \lambda_0 - \frac{v_s \lambda_0}{v_w} = \lambda_0\left(1 - \frac{v_s}{v_w}\right)$$

↑ velocity of source; v_w ← wave velocity

$$\boxed{\lambda_{left} = \lambda_0(1 + v_s/v_w)}$$

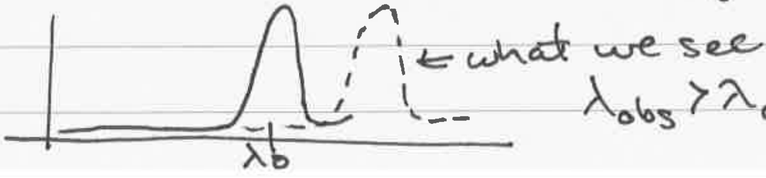
DEMONSTRATION

Sonic boom ⇒ wave fronts very tightly spaced... lots of energy packed into wave near nose of supersonic jet or other device

Utility of doppler Cosmology

we know distant galaxies are receding from us, universe is expanding

Normal modes of oscillation of atoms ⇒ absorbtion or emission lines in light spectra



$\lambda_{obs} > \lambda_0$ ⇒ object moving away
"redshifted"