Lecture 12.2

→ Energy and momentum in mechanical waves (from 12.1 notes)

→ Boundary effects / Interference

→ Reflected and transmitted waves

I. Wave reflection

→ In our systems of finite size, boundary conditions set the spectrum of normal modes

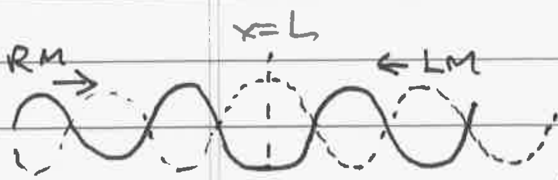
→ furthermore, normal modes can be written as superposition of LM+RM traveling waves

Recall → LM + RM waves conspire to always have destructive interference at fixed ends of string (modes of standing wave)

Standing wave (on fixed string) $y(x,t) = A\sin\left(\frac{n\pi x}{L}\right)\cos(\omega t)$

$$= \frac{A}{2}\Big[\underbrace{\sin\left(\frac{n\pi x}{L} - \omega t\right)}_{RM} + \underbrace{\sin\left(\frac{n\pi x}{L} + \omega t\right)}_{LM}\Big]$$

@ $x = 0, L$ $\quad y(0,t) = y(L,t) = \frac{A}{2}\sin(-\omega t) + \frac{A}{2}\sin(\omega t)$



always vanishes!

always equal & opposite

★ Now consider wave pulses

②

→ Image on page 255 of text
* Show on computer screen

By Newton's 3rd law: incident pulse exerts force on fixed end, end must exert equal & opposite force!

* generates pulse of opposite polarity!

For open end, tension component in vertical direction vanishes → force added to cancel force exerted by pulse → pulse of same polarity!

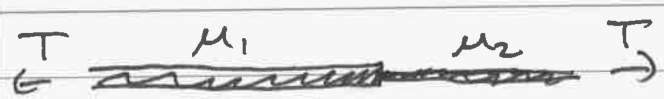
~~Fixed reflection: fixed end~~

~~$x = 0$~~

~~under $x \to -x$, get a minus sign~~

"Refraction" on strings

Consider: 2 strings w/ different mass per-length μ_1 and μ_2 joined together



What happens to waves at this boundary

③

* Bell Labs demo

Outcome - Transmitted wave + reflected wave!

Imagine string to be very long so no reflection from endpoints, only at junction

→ Write solution to wave equation w/pulse sent to the right:

Region 1:

$$y_1(x,t) = f_1\overbrace{\left(t - \frac{x}{v_1}\right)}^{\text{note this form is equivalent to } x \pm vt} + g_1\overbrace{\left(t + \frac{x}{v_1}\right)}$$

↑ wave we sent ↑ reflected wave at boundary

Region II

$$y_2(x,t) = f_2\left(t - \frac{x}{v_2}\right)$$

↑ transmitted wave

In I: $\dfrac{\partial^2 y_1}{\partial x^2} = \dfrac{1}{v_1^2}\dfrac{\partial^2 y_1}{\partial t^2}$ II: $\dfrac{\partial^2 y_2}{\partial x^2} = \dfrac{1}{v_2^2}\dfrac{\partial^2 y_2}{\partial t^2}$

Take strings to be joined at $x=0$:

$$y_1(x=0,t) = y_2(x=0,t)$$

also so acceleration is not discontinuous

$$\left.\frac{\partial y_1}{\partial x}\right|_{x=0} = \left.\frac{\partial y_2}{\partial x}\right|_{x=0}$$

So $$f_1(t) + g_1(t) = f_2(t)$$

and $$\frac{1}{v_1}\left(-f_1'(t) + g_1'(t)\right) = \frac{-1}{v_2} f_2'(t)$$

$$\Downarrow \text{ integrate and rearrange}$$

$$(f_1 - g_1) = \frac{v_1}{v_2} f_2$$

Now can <u>solve</u> for f_2 and g_1! (transmitted/reflected waves)

$$g_1(t) = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) f_1(t)$$

$$f_2(t) = \left(\frac{2v_2}{v_1 + v_2}\right) f_1(t)$$

replace x dependence so it all satisfies wave eq.

$$g_1\left(t + \frac{x}{v_1}\right) = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) f_1\left(t + \frac{x}{v_1}\right)$$

$$f_2\left(t - \frac{x}{v_2}\right) = \left(\frac{2v_2}{v_1 + v_2}\right) f_1\left(t - \frac{x}{v_2}\right)$$

Same shape, but rescaled in height and width