$\boxed{\text{Lecture 11.1}}$ Progressive / Traveling Waves

→ Thus far, in "continuous" media, we have focussed on normal mode solutions

* Any excitation is sum over normal modes

→ There is a special class of excitations corresponding to pulses with short duration

(Short ⟹ $\Delta t \ll \dfrac{L}{v}$ L → size of system, v → wave velocity)

→ can convey <u>information</u> over distances

* Would be wonderful to have formalism for these!

I. <u>from normal modes to traveling waves</u>

→ Normal modes can be written as sum of left- and right-moving waves

<u>String:</u> 0 |——| L (fixed ends)

<u>Normal mode</u> $y_n(x,t) = A_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t)$

trig relation: $\sin\theta\cos\phi = \frac{1}{2}\left[\sin(\theta-\phi) + \sin(\theta+\phi)\right]$

$$\Rightarrow \; y_n(x,t) = \frac{A_n}{2}\Big[\underbrace{\sin\left(\frac{n\pi x}{L} - \omega_n t\right)}_{\text{right-moving}} + \underbrace{\sin\left(\frac{n\pi x}{L} + \omega_n t\right)}_{\text{left-moving}}\Big]$$

(right moving → as t grows, x must increase to keep function same
left moving → as t grows, x must decrease to keep function same

②

Since $\omega_n = \frac{n\pi v}{L}$, can rewrite as

$$\sin\left(\frac{n\pi x}{L} \mp \omega_n t\right) = \sin\left[\frac{n\pi}{L}(x \mp vt)\right]$$

can generalize further with $\lambda_n = \frac{2L}{n}$

$$y(x,t) = \cancel{A}\sin\left[\frac{2\pi}{\lambda}(x \mp vt)\right]$$ is it a solution to wave equation? YES (easy to check)

Satisfies $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

* Not a new solution - different ways of talking about solutions to wave equation

→ Note that Boundaries are Important

→ Original normal mode $$y_n(x,t) = A_n \sin\left(\frac{2\pi x}{\lambda_n}\right)\cos\left(\frac{n\pi v t}{L}\right)$$

$$= \frac{A_n}{2}\left\{\sin\left[\frac{2\pi}{\lambda_n}(x - vt)\right] + \sin\left[\frac{2\pi}{\lambda_n}(x + vt)\right]\right\}$$

@ $x = 0$, $\sin\left(\frac{-2\pi v t}{\lambda_n}\right) + \sin\left(\frac{+2\pi v}{\lambda_n}t\right) = 0$

@ $x = L$ $\sin\left(n\pi - \frac{2\pi v t}{\lambda_n}\right) + \sin\left(n\pi + \frac{2\pi v t}{\lambda_n}\right) = 0$ also

[At nodes of normal mode / standing wave, Left + Right moving waves interfere destructively (always!)

Anti-nodes - L + R waves interfere constructively

③

Let us now focus on just right moving waves
→ To facilitate → take L very large (ignore reflections)

(DEMO – IBM torsion rod waves)

Right-moving wave: $y(x,t) = A \sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$

↳ Any wavelength (not rest to normal mode)

i.e. from driving string @ $x=0$ with simple harmonic motion

Now consider driving for some time $T = t_2 - t_1$, then stopping

front of wave moves by $x_1 = v(t - t_1)$

back moves by $x_2 = v(t - t_2)$

so $x_1 - x_2 = v(t_2 - t_1)$

★ Information (how long person was shaking) transmitted down string

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For pure sinusoidal motion:

$v = \lambda f$ and v is assumed constant

(ie $v = \sqrt{T/\mu}$, independent of λ)

More generally, $v = v(f)$ "dispersion relation"

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Wave speeds

→ i) String or wire: $v = \sqrt{\frac{T}{\mu}}$

i.e. $T = 100N$, $\mu = 0.5 g/m$ ⇒ $\boxed{v = \sqrt{\frac{100N}{.5 g/m}} = 450 m/s}$

DEMO – long green fabric rope – adjust tension, watch wave speed change

→ ii) Sound waves in gas columns

$$v = \sqrt{\frac{K_{adiabate}}{\rho}} \quad \text{and} \quad K_{adia} = \underset{\text{1.667 ideal, 1.4 air}}{\gamma} p$$

air, $\rho = 1.2 kg/m$, $p \cong 100{,}000 \ N/m^2$

$$\boxed{v = \sqrt{\frac{100{,}000 \ N/m^2 \cdot (1.4)}{1.2 \ kg/m}} \simeq 340 \ m/s}$$

SUPERPOSITION

Progressive / Traveling waves

→ The wave equation is linear so superposition principle holds (traveling or standing waves)

i.e. superposition of 2 right-moving waves of same amplitude, different wavelength:

$$y_1 = A\sin\left(\frac{2\pi}{\lambda_1}(x - vt)\right) \qquad y_2 = A\sin\left(\frac{2\pi}{\lambda_2}(x - vt)\right)$$

$$y = y_1 + y_2 = A\left[\sin\left(\frac{2\pi}{\lambda_1}(x - vt)\right) + \sin\left(\frac{2\pi}{\lambda_2}(x - vt)\right)\right]$$

use usual "beat" formula!

(5)

$k = 1/\lambda$ (sometimes $2\pi/\lambda$ in some contexts)

define ~~k~~ (wave-number)

$$y = A\left[\sin(\cancel{2\pi}k_1(x-vt)) + \sin(\cancel{2\pi}k_2(x-vt))\right]$$

$$= \frac{A}{2}\sin\left[2\pi\underbrace{\left(\frac{k_1+k_2}{2}\right)}_{k_{avg}}(x-vt)\right]\cos\left[2\pi\underbrace{\left(\frac{k_1-k_2}{2}\right)}_{\frac{\Delta k}{2}}(x-vt)\right]$$

Beats in time @ fixed x, and in ~~space~~ x at fixed time

wave pattern moving in time to right

$$D = \frac{1}{k_1-k_2} = \frac{1}{\frac{1}{\lambda_1}-\frac{1}{\lambda_2}} = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}$$

DEMO → TUNING FORKS AGAIN

NEXT TIME → Wave pulses