$\boxed{\text{Lecture 3.2}}$ Announcements 1) HW regrade → turn assignments back in

2) Colloquium – Observation of early universe gravity waves in the pattern of very old E.M. waves (CMB)

To begin, let us work through an exercise:

Solve the equation $a\ddot{x}+b\dot{x}+cx=0$ → guess solution of form $x=Ae^{pt}$

Ans: $x=A_+e^{\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)t}+A_-e^{\left(-\frac{b}{2a}-\sqrt{b^2-4ac}\right)t}$ Where does this come from?

$\dot{x}=\frac{d}{dt}Ae^{pt}=pAe^{pt}=px$ $\ddot{x}=p^2Ae^{pt}=p^2x$ → Replace in differential equation: $ap^2\cancel{x}+bp\cancel{x}+c\cancel{x}=0$

$ap^2+bp+c=0$ Differential Equation →

2 Solutions $p_\pm=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Quadratic formula (can be complex!)

Now we utilize superposition principle (works because this is a linear differential equation)

$x_+=A_+e^{p_+t}$ $x_-=A_-e^{p_-t}$ and $x=x_++x_-$ is also solution

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We can apply this technique to the scenario where there is a damping force proportional to velocity

Two components to force: $F=-kx-b\dot{x}$ Can't be obtained via $F=-dU/dx$

conservative — Energy remains in system

dissapative — Energy given to surroundings

Newton's 2nd $F=-kx-b\dot{x}=m\ddot{x}$, or $m\ddot{x}+b\dot{x}+kx=0$ ⇒ $x(t)=A_+e^{-\frac{b}{2m}t}e^{+i\omega t}+A_-e^{-\frac{b}{2m}t-i\omega t}$ with $\omega=\sqrt{k/m-\left(\frac{b}{2m}\right)^2}$ (for small b)

Let us spend some time talking about damped systems in more detail

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Solutions from $p_\pm=\frac{-b}{2m}\pm i\sqrt{\frac{k}{m}-\left(\frac{b}{2m}\right)^2}$

$\boxed{\text{3 Cases}}$ 1) p is complex $\left(\frac{b}{2m}\right)^2<\frac{k}{m}$ stuff in $\sqrt{\ }$ is positive BEHAVIOR= oscillations w/ exponentially decaying amplitude

$x(t)=Ae^{-\frac{b}{2m}t}\cos(\omega t+\alpha)$ (Real part of general complex sol'n)

2) p is real $\left(\frac{b}{2m}\right)^2>\frac{k}{m}$ Stuff in $\sqrt{\ }$ is negative PURE EXPONENTIAL DECAY

$x(t)=A_+e^{\left(-\frac{b}{2m}+\sqrt{\left(\frac{b}{2m}\right)^2-\frac{k}{m}}\right)t}+A_-e^{\left(-\frac{b}{2m}-\sqrt{\left(\frac{b}{2m}\right)^2-\frac{k}{m}}\right)t}$ "OVER DAMPED"

↳ slower decay ↳ faster decay

3) P is real (but just barely) $\left(\frac{b}{2m}\right)^2=\frac{k}{m}$ Stuff in $\sqrt{\ }$ is zero pure exponential decay

$x(t)=Ae^{-\frac{b}{2m}t}$ "Critically damped" Excitation gone fastest

$\boxed{\text{Forced (or "driven") Oscillations}}$ Many systems are driven externally. Energy may be lost to dissipative forces, but at equilibrium, inflow = outflow