Lecture 3.1

"Oscillations are everywhere" Off-diagonal terms in $U(x,y)$ B-meson oscillation

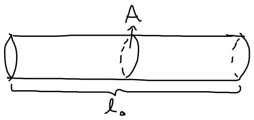
Two Crucial Features: 1) Inertial component → Carries kinetic energy "Mass"
2) Elastic component → Stores elastic potential energy "Spring"

} These are much more general than just mass on spring

Example Mass m on finite mass spring w/ mass per length λ, length L (actually quite complicated to analyze in detail!)

Example Vibrations of semi-rigid bodies

Consider rod (or wire) w/ length l_0, cross-sectional area A:



Consider stretching rod/wire by $\Delta l \ll l_0$ (Applying force ΔP to end)

"Strain" $\equiv \Delta l / l_0$

"Stress" $\equiv \frac{\Delta P}{A}$ units of pressure

For small displacements (strain $= \frac{\Delta l}{l_0} \ll 1$) $\frac{\text{stress}}{\text{strain}} = \text{constant}$ Property of Material!

↑ Young's Modulus of Elasticity

Equation for stretching of material $\frac{(dF/A)}{(dl/l_0)} = -Y$, or $dF = -\frac{AY}{l_0} dl \Rightarrow$ for uniform stretching by amount x: $F = -\frac{AY}{l_0} x$ like Hooke's Law!

Note: This fails @ strains larger than ~ 0.1% 1) non-linear terms
2) wire breaks

Analyzing oscillations: Hang mass m on wire → $F = -\frac{AY}{l_0} x = m\ddot{x} \Rightarrow \ddot{x} = -\frac{AY}{l_0 m} x \Rightarrow \omega^2 = \frac{AY}{l_0 m}$ $\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{l_0 m}{AY}}$

for $l_0 = 1m$, $m = 1kg$

steel w/ diameter 1mm $T = 1.6 \times 10^{-2}$ sec
$f = 60Hz$

DEMO- TORSION PENDULUM – Also book examples

1) Floating objects $m\ddot{y} = -g\rho A y$ ↱ buoyant force Archimedes!
2) Complicated pendula $I\ddot{\theta} = -mgh\theta$ ↑dist. to CM
3) Water in tube $\rho A l \ddot{y} = 2g\rho A y$

} Trick: I.D. the KE and PE of system conserve E_{tot}, take derivative!

In-class exercise: Show that Ae^{pt} solves $m\ddot{x} = -kx$ if $p = i\omega = i\sqrt{\frac{k}{m}}$

Hint $\frac{d}{dx} e^{\lambda x} = \lambda e^{\lambda x}$

Now, what is p for $m\ddot{x} + b\dot{x} + kx = 0$?

Damping In nearly all cases, oscillating system eventually lose energy to the environment (heat, other waves, etc.)

Motion of body through fluid → Resistive force $R(v) = \underbrace{\boxed{bv}}_{\hookrightarrow \text{first non-trivial term}} + cv^2 + \cdots$ for small v

Newton's Law $F = -kx - b\underset{\substack{\uparrow \\ bv}}{\dot{x}} = m\ddot{x}$ Exactly the math example you just worked out! $x \to z(t) = A e^{pt}$

$$mp^2 \cancel{x} + bp\cancel{x} + k\cancel{x} = 0$$

$$p = \frac{-b \pm \sqrt{b^2 - 4km}}{2m} = -\frac{b}{2m} \pm i\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

Solution $z(t) = A_+ e^{\left(-\frac{b}{2m} + i\omega\right)t} + A_- e^{\left(-\frac{b}{2m} - i\omega\right)t}$ Oscillations w/ exponentially decaying amplitude!

Equiv. $x(t) = \mathrm{Re}\left[A e^{-b/2m\, t} e^{i(\omega t + \alpha)}\right]$