Lecture 2.2 Last time: We explored the phenomenon of superposition: When a physical system is linear (equations only involve $x, \dot{x}$, not x^2...)

SUM of solutions = Solution

CRUCIAL for Wave propagation

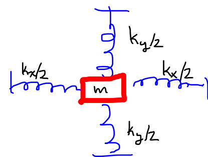
Superposition principle ⟷ Interference phenomena
BEATS, Interference patterns, iridescence

} So far we have examined 1D superposition (ie. effects of 2 superposed sound waves on eardrum)

Phenomena of 2 (or more) degrees of freedom far richer!

Show video of bowling ball pendula → Jeff & Margot in NC (ON website) Most systems have many ways in which they move!

I. 2 vibrations ⊥ to each other:



Mass moves (oscillates) in both directions

$$F_x = -k_x x = m\ddot{x} \qquad F_y = -k_y y = m\ddot{y}$$
$$\omega_x = \sqrt{\frac{k_x}{m}} \qquad \omega_y = \sqrt{\frac{k_y}{m}}$$

Aside: MATRICES ARE USEFUL!

$$\vec{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = -\hat{k}\vec{x} = -\begin{pmatrix} k_x & 0 \\ 0 & k_y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= -\begin{pmatrix} k_x x \\ k_y y \end{pmatrix} = m\ddot{\vec{x}} = m\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$$

Compact way to represent complicated phenomena!

Solutions 2 identical diff. eq.s to integrate / solve

X: $x(t) = A_x \cos(\omega_x t + \alpha_x)$ $\qquad y(t) = A_y \cos(\omega_y t + \alpha_y)$

or $x(t) = \mathrm{Re}[A_x e^{i(\omega_x t + \alpha_x)}]$ $\qquad y(t) = \mathrm{Re}[A_y \cos(\omega_y t + \alpha_y)]$

What are the phenomena of such 2D systems? (Also UBIQUITOUS): 2D $U(x,y)$, near minimum! $U_{springs} = \frac{1}{2}k_y y^2 + \frac{1}{2}k_x x^2$

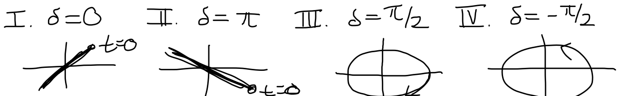
II. 2D oscillations, same frequency!

$x(t) = A_x \cos(\omega t + \alpha_x)$
$y(t) = A_y \cos(\omega t + \alpha_y)$ } $x(t) = A_x\cos(\omega t)$
$y(t) = A_y\cos(\omega t + \delta)$

Depending on relative phase, different phenomena possible

Figure 2-10 in textbook

I. $\delta = 0$ II. $\delta = \pi$ III. $\delta = \pi/2$ IV. $\delta = -\pi/2$



Lines!
$x = \pm\frac{A_x}{A_y} y$

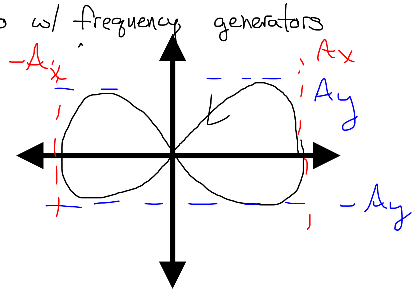
$x = A_x\cos(\omega t)$
$y = A_y\sin(\omega t)$

ellipse!
$$\frac{x^2}{A_x^2} + \frac{y^2}{A_y^2} = \cos^2(\omega t) + \sin^2(\omega t) = 1$$

III. 2D oscillations, 2 frequencies Demo w/ frequency generators

$x(t) = A_x\cos\omega t \qquad y(t) = A_y\sin(2\omega t)$

$-\pi/2$ phase shift



Show in Mathematica

Aside by far, coupled oscillators are the most interesting!
I. Willberforce pendulum
II. 32 metronomes

Now for a return to the SHO (and beyond)

There are always 2 ways to write the equations for a mech. system 1) Newton's Laws $-kx = ma = m\ddot{x}$

2) Conservation of Energy $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E_{total}$ (just integral of 1st

$\frac{1}{2}m(\dot{x})^2 + \frac{1}{2}kx^2 = E_{tot}$

Let us study a method of solving such equations starting with complex exponentials

$\ddot{x} = -\omega^2 x$ Now guess solution has form $x = Ce^{pt}$ and plug in... $\ddot{x} = p^2 Ce^{pt} = -\omega^2 Ce^{pt}$ Works! If $p^2 = -\omega^2$, or $p = \pm\sqrt{(-1)\omega^2} = \omega\sqrt{-1} = i\omega$

$x = C_+ e^{i\omega t} + C_- e^{-i\omega t}$

Damping...