Lecture 1.1

Why vibrations and waves? **Ubiquity:** Communication, energy and information transfer, properties of mechanical structures, circuit design, biological rhythms, etc..... impossibly long, semi-infinite list

Characterized by PERIODICITY/repetition

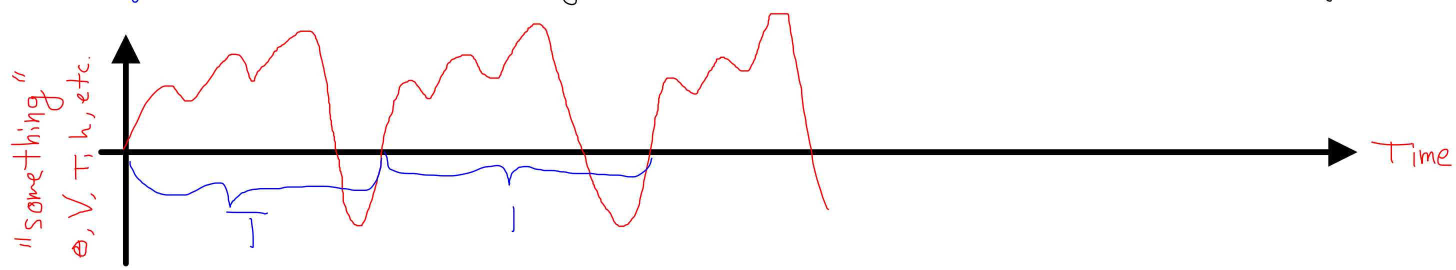
Example Electric guitar

1) electro-chemical impulse travels from brain to fingers/musculature
2) string vibrates in a multitude of frequencies, one dominant "note" + "harmonics"
3) moving string creates disturbances in magnetic field of magnets embedded in coils
4) Changing B field creates E-field near coils, in turns creates pulses of current that travel to amplifier (powered by sinusoidally varying current)
5) amplified current signal turned back into $\vec{B}$ field on speaker solenoid, creates mechanical force on speaker "plate" (also E.M. waves that propagate into space)
6) vibrating speaker disturbs air, sound waves travel and spread (most energy in wave becomes heat in walls = vibrations of molecules in solids)
7) your eardrum vibrates in response, sending another electrochemical impulse to your brain

"HW problem": Sit quietly, try to follow/meditate on wave phenomena around you ☺

I. Vibratory Phenomena

→ essentially, something that repeats, w/ period T, i.e. rotation of planet through one orbit, EKG cycle, AC power

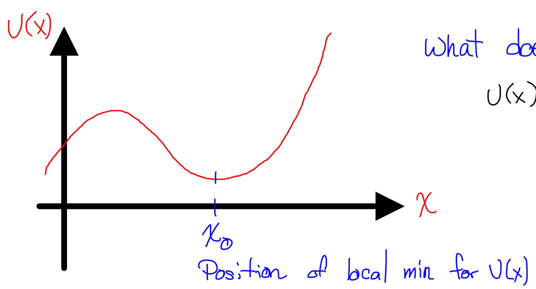


For a while, we occupy ourselves with pure sinusoidal oscillation (not just an approximation... all periodic motion can be built from pure sinusoids)

Also ALL SINGLE DEGREE OF FREEDOM SYSTEMS CLOSE TO EQUILIBRIUM OSCILLATE AS PURE SINUSOIDS

Let us explore this last statement Newton's 2nd Law: $F = -\frac{dU}{dx} = m\ddot{x}$ Why is sinusoidal (a.k.a. Simple Harmonic) motion ubiquitous?

A generic potential:



What does U(x) look like near x_0? TAYLOR EXPAND

$$U(x) \simeq U(x_0) + \frac{dU}{dx}(x_0)(x-x_0) + \frac{1}{2}\frac{d^2U}{dx^2}(x_0)(x-x_0)^2 + \cdots$$

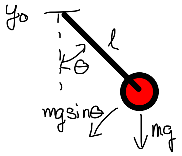
0 at minimum! ← positive at minimum

Apply Newton's 2nd: $F = -\frac{dU}{dx} = -\frac{d^2U}{dx^2}(x_0)\,(x-x_0) = m\ddot{x}$

OR $m\ddot{\Delta x} = -k\,\Delta x$ HOOKE'S LAW

EVERYTHING IS A MASS ON A SPRING

Example Ye noble pendulum



$F = -mg\sin\theta = m\ddot{x}$

$-mg\theta = m\ddot{x}$

$-(mg/\ell)x = m\ddot{x}$

★ Note x is length of chord on circle, not distance from axis

Or: $U = mgh = mg(y_0 - \ell\cos\theta) \simeq mg(y_0 - \ell) + \frac{1}{2}mg\ell\theta^2 + \cdots = mg(y_0-\ell) + \frac{1}{2}\left(\frac{mg}{\ell}\right)x^2 + \cdots$

$\hookrightarrow \simeq 1 - \frac{\theta^2}{2} + \cdots$ $\quad \uparrow x = \ell\theta$

So $F = -\frac{dU}{dx} = -mg\ell\sin\theta \simeq \left(\frac{mg}{\ell}\right)x \Big\} = m\ddot{x}$

Moral All 1DoF Systems near equilibrium are S.H.O.'s and have nearly pure sinusoidal motion!

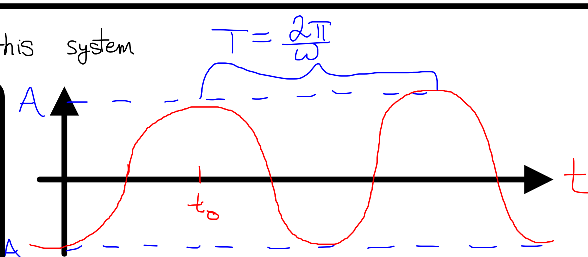
This Motivates: Build a toolkit to solve these systems very efficiently!! THE COMPLEX EXPONENTIAL

I. Periodic Motion

The "mass-on-a-spring" is in fact extraordinarily general, so specify to this system

$F = -kx = m\ddot{x} \Rightarrow \ddot{x} = -\frac{k}{m}x = -\omega^2 x \quad \left(\omega = \sqrt{\frac{k}{m}}\right)$

$$\boxed{\begin{array}{l}\text{Complete sol'n: } x(t) = A\cos(\omega(t-t_0)) \\ \text{often } x(t) = A\cos(\omega t + \alpha) \\ \qquad\qquad \hookrightarrow \alpha = -\omega t_0 \\ A: \text{"Amplitude"} \\ \alpha: \text{"phase"}\end{array}}$$



PROBLEM Trigonometric functions are cumbersome, identities abstruse, and generally demoralizing

IMPORTANT LESSON At times, the addition of extraneous / seemingly irrelevant info/stuff improves our situation (i.e. packing peanuts)

II. Rotating vectors – guided example

FIRST, DEMO = spring + motorized wheel

I. Projection of vector, length A, onto $\hat{x}$ component, angle θ wrt. x-axis

$\hat{y}$ component?

II. parametrize a rotating vector, rotating w/ constant angular velocity ω, as a function of time. Assume angle, at time $t=0$, is α

III. $e^{i\pi} + 1 = 0$

The complex plane carries the same information as 2D vector space $z = x + iy \rightarrow z = Ae^{i(\omega t+\alpha)} = A\cos(\omega t+\alpha) + iA\sin(\omega t+\alpha)$

$\hookrightarrow$ real #

Motivate by Taylor series:

$\cos\theta \simeq 1 - \frac{1}{2}\theta^2 + \frac{\theta^4}{4!} - \cdots$

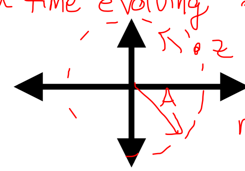
$\sin\theta \simeq \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5} - \cdots$

combine: $\cos\theta + i\sin\theta = 1 + \frac{i\theta}{1!} - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} - \cdots$

$= \frac{(i\theta)^0}{0!} + \frac{(i\theta)^1}{1!} + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \cdots \simeq e^{i\theta}$!!!

Rotating vector $\longrightarrow$ Complex number with time evolving phase!

$\boxed{\text{Physics is in the real part}}$ $\quad x(t) = \mathrm{Re}[z(t)] = A\cos(\omega t + \alpha)$



rotates with angular velocity ω

IV. Why do we care? Is the extra imaginary baggage worth it? In part, time will tell, but generally speaking:

EXPONENTIALS ARE EASY

1) Taking derivatives: Often interested in velocities, accelerations: $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$:

$$x(t) = A\cos(\omega t+\alpha)$$
$$\dot{x}(t) = -A\omega\sin(\omega t+\alpha)$$
$$\ddot{x}(t) = -A\omega^2\cos(\omega t+\alpha)$$

Using exponentials: $z(t) = Ae^{i(\omega t+\alpha)}$

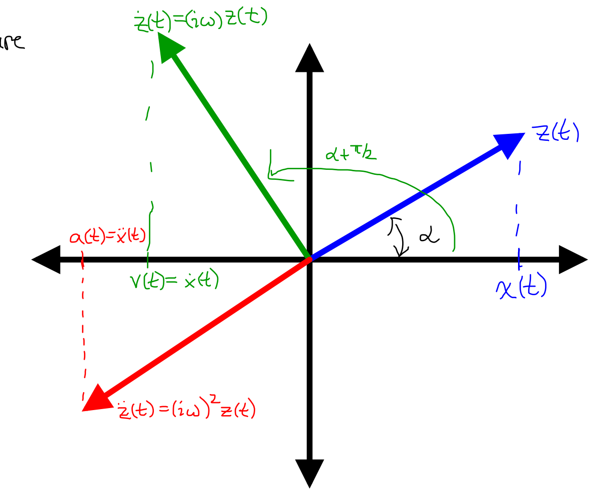
$$\dot{z}(t) = A(i\omega)e^{i(\omega t+\alpha)} = A\omega e^{i\pi/2}e^{i(\omega t+\alpha)} = A\omega e^{i(\omega t+\alpha+\pi/2)} = i\omega z$$

$$\ddot{z}(t) = A(i\omega)^2 e^{i(\omega t+\alpha)} = A\omega^2 e^{i\pi}e^{i(\omega t+\alpha)} = A\omega^2 e^{i(\omega t+\alpha+\pi)} = (i\omega)^2 z$$

$$\frac{d^n}{dt^n}z(t) = A(i\omega)^n e^{i(\omega t+\alpha)} = A\omega^n e^{in\pi/2}e^{i(\omega t+\alpha)} = A\omega^n e^{i(\omega t+\alpha+\frac{n\pi}{2})} = (i\omega)^n z$$

easy to see derivatives have "advanced phase"

2) geometrical picture



Will see on thursday that there are benefits for superposition as well