Lecture 11.1

Last time- derived propagator $\langle 0| TEA_{\mu}(x) A_{\nu}(y) 3 |0 \rangle$

Issues were:
1. Gauge invariance makes kinetic operator non-invertible
2. Add gauge fixing term: $\frac{1}{2\xi} (\partial_{\mu} A_{\nu} - \frac{1}{2} \delta_{\mu\nu} \partial_{\rho} A^{\rho})^2$
   $\Rightarrow$ Why does this work? We just break gauge invariance!
3. New kinetic operator is invertible

$$i\Pi^{\mu\nu} = \frac{i}{p^2 + i\varepsilon} \left[ g^{\mu\nu} - (1-\xi) \frac{p_{\mu} p_{\nu}}{p^2} \right]$$

* Physics is independent of $\xi$!* *

Note can also perturbative

Physics just in terms of $F_{\mu\nu}(E+B)$, but looks very non-local i.e. $(\frac{1}{D})_{\mu\nu} F_{\mu\nu}$ in interacting

$\Rightarrow$ Will skip higher spin except to discuss role of gauge invariance

Spin-2: field $h_{\mu\nu}$ (gravitons are excitations)

Massless $\Rightarrow$ require a gauge invariance (↔ diffeomorphism)

G.T.: $h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_{(\nu} q_{\mu)} + \partial_{\mu} q_{\nu}$

$\Rightarrow$ alpha

Very restrictive on form of kinetic term:

$$\mathcal{L}_{\text{kin}} = \frac{1}{4} h_{\mu\nu} \partial^{\mu\nu} + \frac{1}{2} h_{\mu\nu} \partial_{\mu} q_{\nu} + \frac{1}{2} h_{\mu\nu} \partial_{\nu} q_{\mu} - \frac{1}{2} h_{\mu\nu} q_{\mu\nu} - \frac{1}{4} h_{\mu\nu} q_{\mu\nu}$$

Fierz-Pauli kinetic term (matches E-H action)
Scalar QED

1. Complex scalar field - recall Aμ needs to couple to conserved current, Jμ, and single scalar field has no symmetries to associate with gauge redundancy

\[ \phi : \quad (\partial^2 + m^2) \phi = i(\partial A_\mu) \phi \partial^\mu + i \partial_\mu (\partial A_\mu) A^\mu \phi + (-eA_\mu) \phi \]

\[ \phi^* \text{ eq. in h.c. (Note } e \phi^* = -e \phi) \]

Opposite charge, same mass (antiparticles)

Recall real quantum scalar field:

\[ \hat{\phi} = \int \frac{d^3p}{(2\pi)^3 \sqrt{2m}} (\hat{a}_p e^{-i p x} + \hat{a}^+_p e^{i p x}) \]

\[ \hat{\phi}^+ = \hat{\phi} \]

Now for C scalar field, \( \phi \neq \phi^* \)

\[ \Rightarrow \hat{\phi} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2m}} \left[ \hat{a}_p e^{-i p x} + \hat{b}_p e^{i p x} \right] \]

\[ \hat{b}_p \text{ 's create & annihilate particles of same mass} \]

Can calculate \( Q = \int d^3x J^\mu \) → find \( b \)'s create opposite change particles

Inescapable: want \( \phi \) coupling to matter in relativistic manner \( \Rightarrow \) need anti-particles

Feynman Rules

(Expanded \( |D_\mu \phi|^2 \))

\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu}^2 - \frac{1}{2} \partial_\mu \phi (D^\mu + m^2) \phi - i e A_\mu [\phi^* \phi , \phi - (\partial \mu \phi) \phi] \]

\[ + e^2 A_\mu^2 |\phi|^2 \]

\[ \text{arrows indicate change of } \phi \text{ as } \phi \leftrightarrow \phi^* \text{ w/ } \phi \text{ and } \phi^* \text{ w/ } \phi^* \text{ vanish} \]

Scalar propagator:

\[ \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi^*} = \frac{1}{p^2 - m^2 + i \epsilon} = \langle 0 | T \phi(x) \phi^*(y) | 0 \rangle \]

Photon propagator:

\[ \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi^*} = \frac{-i}{p^2 + i \epsilon} \left[ g^\mu \nu - (1 - \delta^\mu \nu) \frac{p^\mu p^\nu}{p^2} \right] \]
Note that it contains derivatives of $g^{\mu\nu}$ at vertices $\Rightarrow$ factors $g^{\nu\mu}$

assuming $a/3$ are for $e^-$ (scalar electrons) final:

$$e^- \quad p_2 = i e (-p_1^\mu - p_2^\mu)$$

$$e^+ \quad p_2 = i e (p_1^\mu + p_2^\mu)$$

Basic rule: $-\mathbf{p}$ if momentum flows in same direction as charge $+$ $\mathbf{p}$ otherwise

$\Rightarrow$ generally don't write out separate rules

4. Anti-particles $\Rightarrow$ momentum flow against charge flow

Final Rule

$$\epsilon = 2i e^2 g^{\mu\nu}$$

External states nothing new for scalar fields (no polarization info)

Photon (spin-1): $A_{\mu}(\mathbf{x}) = \int \frac{dk}{(2\pi)^3} \frac{1}{4\pi^2} \epsilon^{\nu\rho\sigma} \left[ \epsilon_{\nu}^i(k) a_{\rho\sigma} e^{-ikx} + h.c. \right]$

CZ modified by attaching $\epsilon_{\nu}^i(k)$ for external states

$\epsilon^i_{\mu}$ for incoming $(\epsilon^i_{\mu})^{*}$ for outgoing

Example: "electron" photon scattering

$$i\mathcal{M} = \frac{i}{4} \left( p_1 + p_3 \right) \left( p_1 + p_3 \right)^{\nu} \left[ -i\epsilon(2p_2 + p_1)^{\mu} \frac{i}{(p_1 + p_2)^2 - m^2} \right]$$
Scalar Møller Scattering $e^- e^- \rightarrow e^- e^-$ (Example)

2 diagrams $t + u$ channel (s-channel does not conserve charge)

$$i M_t = \frac{e^2}{\pi} \left( p_1 + p_3 \right)^2 \left( -\frac{1}{t} \right) \left( \frac{t}{k^2 + i\epsilon} \right) \left( \frac{1}{u} \right) \left( -\frac{1}{k^2} \right) (-i\epsilon)(p_2 p_4)$$

$$k = p_1 - p_3 = p_2 - p_4$$

Simplifies to $$M_t = \frac{e^2}{\pi} \left( p_1 + p_3 \right) \cdot \left( p_2 + p_4 \right)$$ \& No S-dap

*Note S-dap does not always vanish in each diagram may cancel after summing

$$M_u = \frac{e^2}{\pi} \left( p_1 + p_4 \right) \cdot \left( p_2 + p_3 \right)$$

$$\Rightarrow \frac{d\sigma}{dP} = \frac{1}{64\pi^2 E_m^2} e^4 \left[ \frac{S-U}{t} + \frac{S-T}{u} \right]^2 = \frac{\alpha^2}{4\pi} \left( \frac{E_m}{\sqrt{t}} \right)^2$$

Ward Identity (Example) $e^+ e^- \rightarrow \gamma\gamma$ annihilation

Recall gauge invariance requires $\left| D_{\mu}\phi \right|^2$ and thus both cubic and quartic interactions

Diagrams

\[ \text{Ward Identity } e \rightarrow \gamma \Rightarrow P_\mu M^\mu = 0 \]

need all three diagrams for this to hold,
Can prove in general that any 5* amplitude in scalar QED satisfies the Ward identity and S-independence.

Next time Weinberg soft theorems ⇒ constraints on couplings of massless spin 1 and spin 2 particles.

Starting on Spinor Fields