Last time Massive spin-1  
1) Most general \( F \) has just 4 scalars 
2) Restrict so only 3 dof: \( F_{\mu\nu} \) 
3) 3 pol. vectors, 2 transverse, 1 long. 
4) long: scattering badly behaved 

Massless Spin-1 and Gauge invariance

\[ m \to 0 \text{ limit } \quad \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 \]

Issues:  
1) \( m^2 (\partial \mu A^\mu) = 0 \) imposes no constraint  
2) \( \varepsilon_\mu = (R^\mu_\mu 0, 0, 0, E) \to \infty \) 
3) Should only have 2 \( \to \) seem to have 4 again

Note that \( F \) has a gauge invariance:

\[ A_\mu \to A_\mu + \partial_\mu \alpha(x) \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_\mu A_\nu + \partial_\nu A_\mu \]

Under any function \( \partial_\mu A_\nu + \partial_\nu A_\mu \]

\[ = F_{\mu\nu} \]

\[ \Rightarrow 2 \text{ field configurations for some physics} \]

\[ \square A_\mu + \partial_\mu \partial_\nu A^\nu = 0 \to \begin{cases} \partial A_0 + \partial_0 (\partial A_0) = 0 \\ \partial A_i - \partial_0 (\partial_0 A_0 - \partial_i A_i) = 0 \end{cases} \]

\[ \Rightarrow \square A_0 = 0 \text{ and } \square A_i - \partial_0 (\partial_0 A_0 - \partial_i A_i) = 0 \]

\[ \Rightarrow \] no time derivative, and same equation as \( \alpha \)

\[ \Rightarrow \text{can choose } \alpha \text{ to eliminate } A_0 \]

\[ \text{and we still have constraint } \partial_0 A_i = 0 \]

\[ \text{So have } A_0 = 0, \text{ and } \vec{p} \cdot \vec{A} = 0 \]

\[ \text{Eq. } p^\mu = (E, 0, 0, E) \Rightarrow E_0 = 0 \]

\[ \varepsilon_1 = (0, 1, 0, 0) \quad \varepsilon_2 = (0, 0, 1, 0) \]

\[ \varepsilon_3 = 0 \]

\[ \text{Sample basis} \]
"Little group" consider $p^\mu$ fixed \( \rightarrow \) i.e. $p^\mu = (m, 0, 0, 0)$ (massive)

\[ \rightarrow \) SU(2) x-forms leading this fixed are 3D rotations \( \rightarrow \) $SO(3) \rightarrow SO(4)$, $2J + 1$ d.o.f.

$p^\mu = (E, 0, 0, E) \rightarrow ISp(2) \) spin \) J, 2 d.o.f.

\( \rightarrow \) Symmetries of momentum states determine classification of all particles

Covariant Derivatives cannot break gauge invariance anywhere or "sick" modes come back

* Interactions must respect gauge invariance.

Quantum corrections will affect d.o.f. counting

Example \( \phi (\partial \mu \phi) A^\mu \rightarrow \phi (\partial \mu \phi) A^\mu + \phi (\partial \mu \phi) (\partial^\mu \phi) \) extra term

\( \phi \) must transform as well to compensate!

(only possible x-form of real scalar field)

\( \phi \rightarrow \) complex \( \phi \rightarrow e^{-i\xi(x)} \phi \)

\( m^2 |\phi|^2 \) invariant, derivatives are not

\( \phi \rightarrow e^{-i\xi(x)} \phi \)

now \( (\partial_\mu + i e A_\mu) \phi \rightarrow (\partial_\mu + i e A_\mu + i e \partial^\mu \phi) e^{-i\xi(x)} \phi \)

\[ = e^{-i\xi(x)} (\partial_\mu - i e \partial^\mu A_\mu + i e \partial^\mu \phi) \phi \]

\[ = e^{-i\xi(x)} (\partial_\mu + i e A_\mu) \phi \]

\( \Rightarrow \ D_\mu \phi \rightarrow e^{-i\xi(x)} \frac{D_\mu \phi}{D_\mu} \)

Spin-1 x-spin0 interacting theory

\[ S = -\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 - m^2 |\phi|^2 \]

\( \rightarrow \) contains interaction terms
This is the Lagrangian for scalar QED
(-e) is "electric charge" of \( \phi \)

\[ D_\mu \phi = (\partial_\mu - ieQA_\mu)\phi \]

\( D_\mu \) depends on the field it acts on

* Note could take \( \alpha(x) \to \) constant (global continuous symmetry)

then Noether's theorem applies as we derived it

\[ J_\mu = -i(\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi) - 2eA_\mu \phi^* \phi \]

usual quadratic part part due to interactions

forces relationship between 2 point + 3 point correlators!

Note \[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu} + \text{Kinetic terms} + A_\mu J^\mu \]

for fields

always takes this form

Quantization

\[ A^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_{\delta=1}^{3} (e_\delta^\mu (p) \tilde{a}_\delta^\dagger e^{-ipx} + e_\delta^* (p) a_\delta e^{ipx}) \]

\[ a_\delta^\dagger \]\] \( \phi \) type polarization basis vectors \( \omega_p \) \(+2\) trans.

\[ \langle 0 \mid a^\dagger_\delta (p, e^\dagger_\delta) \rangle = \frac{1}{\sqrt{2\omega_p}} \]

\[ \langle 0 \mid A^\mu (p, e^\dagger_\delta) \rangle = e^{i\phi} e^{-ipx} \]

creates particle at position \( x \) with pol. \( e^\dagger_\delta \)